教育部 111 年公費留學考試試題　109

科目：普通物理　　　　　　　　　　　　　　　　(全一頁)

一、一個邊長為 L、質量為 M 的均質正立方體(無歪斜地)浮在密度為 D_0 的水面上。已知重力加速度為 g。(總分 20 分)

(一) 問正立方體沒入水中的深度 d。(10 分)

(二) 垂直輕壓正立方體，放手後立方體微幅上下振盪，其振盪頻率 f 為何？(10 分)

二、兩根長直竿子互相垂直呈十字型，其結合點設為原點。其中一根竿子沿鉛直方向固定，另一根水平竿子可繞著原點、以垂直竿為轉軸轉動。水平竿子上套著一顆質量為 m 的珠子、可無摩擦地沿竿滑動。(總分 20 分)

(一) 一開始竿子與珠子皆靜止不動，珠子離原點距離為 x_0。時間 t=0 時竿子開始以固定角頻率 ω 轉動。問時間 t 時珠子與原點的距離 $x(t)$。(10 分)

(二) 時間 t 時竿子對珠子的施力量值 $F(t)$為何？(10 分)

三、一水平、絕熱的圓柱型桶裡有一無摩擦、不導熱的活塞。活塞兩側各有體積為 V、壓力為 1 atm、溫度為 T 的理想氣體。加熱左方氣體後，活塞向右壓縮，導致右方氣壓升至 2 atm。問此時：(總分 20 分)

(一) 右方氣體溫度 T_R。(10 分)

(二) 左方氣體溫度 T_L。(10 分)

[註: 次方不是整數的數字可保留其次方而不求其數值]

四、一長直螺線管半徑為 a、導線中的電流為 I、每單位長度線圈數為 n。問：(總分 20 分)

(一) 螺線管中的磁場量值 B。(10 分)

(二) 若電流緩慢隨時間振盪: $I(t)=I_0 \sin \omega t$，則螺線管外距離螺線管中心軸 ρ 處的電場 $\boldsymbol{E}(t)$為何？(10 分)

五、一無窮深位能阱的寬度為2ℓ，阱中一質點的波函數為：(總分 20 分)

$\psi(x) = N[\ell^2 - x^2]$、$x \in [-\ell, \ell]$

(一) 找出歸一化常數 N。(10 分)

(二) 偵測時發現此質點位於基態的機率 P 為何？(10 分)

(試題隨試卷繳回)

教育部111年公費留學考試試題 110

科目：天文物理學　　　　　　　　　　　　(全二頁，第一頁)

※以中文或英文作答均可，評分基準相同。請清楚呈現計算的推導及流程。

※相關常數與參數：

萬有引力常數：$G = 6.67 \times 10^{-11} m^3/kg/s^2$

光速：$C = 3 \times 10^8$ m/s

太陽質量：$M_\odot = 1.989 \times 10^{30}$ kg

1 parsec (pc) = 3.0857×10^{16} m

一、(總分12分)星等

(一)設有一顆星球的光度(luminosity)為L，與觀察者的距離為 d，則其視星等 m 的定義為何？(4分)

(二)如何定義其絕對星等 M？(4分)

(三)如果該星球的 m 與 M 均已知，則 d=？(請以 parsec 為單位作答) (4分)

二、(總分20分)超新星

(一)如何區分 I 型及 II 型超新星？(4分)

(二) I 型超新星又可分為 Ia、Ib、Ic 三類。請問 Ia 型超新星的光譜有何特點？(4分)

(三)請問 Ia 型超新星的前身為何？為何適合作為距離的指標？(4分)

(四)當 Ia 型超新星光度抵達尖峰時 M = -19.2，今有一地面望遠鏡觀測一小時可觀測到 m = 24 的星體，試問利用該望遠鏡觀測一小時可觀測的 Ia 型超新星距離最遠為何？(請以 parsec 或公尺為單位回答)(4分)

(五)請敘述在天文宇宙學上，如何利用 Ia 型超新星來推論宇宙是否加速膨脹。(4分)

三、(總分20分)赫羅圖(Hertzsprung–Russell diagram，簡稱為 H–R diagram)

(一)請根據星球的表面溫度，將星球的光譜型(用英文字母表示)由高至低排列。(4分)

(二)每一光譜型又可細分為10類，以數字0, 1, 2, 3,......, 9 表示，請問太陽的光譜型為何(用英文字母加上數字)表示？其表面溫度為多少？(4分)

(三)何謂赫羅圖(Hertzsprung–Russell diagram，簡稱為 H–R diagram)？(4分)

(四)請簡單畫一個 H–R 圖，並標示主序星，紅巨星，超巨星及白矮星的位置。(4分)

(五)若有一星團成員所構成的赫羅圖，要如何估算星團距離我們觀測者的距離？(4分)

(接下頁)

教育部111年公費留學考試試題 110

科目：天文物理學　　　　　　　　　　　　　　　　(全二頁，第二頁)

四、(總分24分)1916年時德國物理學家史瓦濟(Schwarzschild)根據愛因斯坦的重力方程式推導出質量為M且有球形對稱的星體其解如下：

$$(ds)^2 = \left(1 - \frac{2GM/c^2}{R}\right)(cdt)^2 - \frac{1}{1-\frac{2GM/c^2}{R}}(dR)^2 - (Rd\theta)^2 - (R\sin\theta d\phi)^2$$

此解後來被視為質量為M，但沒轉動也不帶電荷的黑洞。

(一)根據此解，黑洞的史瓦濟半徑(Schwarzschild radius)R_S = ?
(此一小題只需寫出表示式，無需轉換成數字)(5分)

(二)假設有一不帶轉動且不帶電的黑洞，其質量M為太陽的f倍，即$M = fM_\odot$，求該黑洞的R_S = ?(請用公里表示)(5分)

(三)黑洞的事件視界為黑洞的史瓦濟半徑所對應範圍的邊界。在黑洞的熱力學定律中，黑洞事件視界的面積，對應為黑洞的熵。設有兩個黑洞其質量分別為$f_1M_\odot$及$f_2M_\odot$，試證若它們合併為一個黑洞時，其熵不會小於原來兩個黑洞的熵的總和。(5分)

(四)簡述黑洞的奇異定理 (singularity theorem)。(5分)

(五)簡述黑洞的無毛定理 (no hair theorem)。(4分)

五、(總分24分)宇宙模型

(一)試寫下愛因斯坦的重力場方程式(包括宇宙常數Λ)。(4分)

(二)試寫下 Robertson-Walker metric。(4分)

(三)試寫下 Friedmann Equation 包括Λ在內。(4分)

(四)請問因何發現，使得愛因斯坦的宇宙靜止模型(static model of universe)被認為是不對的，人們轉為支持大霹靂模型？(4分)

(五)請問原有的宇宙大霹靂理論雖有許多觀測證據支持，但有哪些主要難題無法解釋？(4分)

(六)在1980年美國物理學家古舒(Guth)提出一個想法稱為暴脹(inflation)，可以解決原先宇宙大霹靂理論的難題，請問宇宙的暴脹何時開始發生，即t = 多少秒時發生？此時宇宙以何形式快速膨脹？(4分)

(試題隨試卷繳回)

教育部111年公費留學考試試題 111

科目：量子物理 （全二頁，第一頁）

物理常數及數學公式:

$\hbar \equiv h/(2\pi) = 1.05 \times 10^{-34}\text{J}\cdot\text{s}$；$e = 1.60 \times 10^{-19}\text{A}\cdot\text{s}$；

$m_e = 9.11 \times 10^{-31}\text{kg}$；$m_p \approx m_n = 1.67 \times 10^{-27}\text{kg}$；

$1/(4\pi\epsilon_0) = 9.00 \times 10^{9}\text{m}^{-3}\cdot\text{kg}^{-1}\cdot\text{s}^{4}\cdot\text{A}^{2}$；$c = 3.00 \times 10^{8}\text{m}\cdot\text{s}^{-1}$。

黎曼 zeta 函數: $\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s}$。 $\zeta(2) = \frac{\pi^2}{6}$; $\zeta(4) = \frac{\pi^4}{90}$; $\zeta(6) = \frac{\pi^6}{945}$。

一、 已知一維簡諧振子的能量為$E = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$。其中 m 為粒子質量、ω為系統自然頻率、q 為位移而 p 為動量。由 Bohr-Sommerfeld 量子化條件，求簡諧振子的量子能階E_n。(15 分)

二、 電子與正子的粒子對產生過程: $\gamma + \gamma \rightarrow e^- + e^+$，電子質量為 m。碰撞前兩個光子(γ)的頻率皆為 ν，行進方向則分別為$\hat{x}$及$-\hat{x}$。碰撞後電子(e^-)的能量為 E，動量則為$p(\cos\theta\,\hat{x} + \sin\theta\,\hat{y})$。下列問題的答案以 m、ν 及相關物理常數表示之。(總分 10 分)

(一)求碰撞後正子(e^+)的能量及動量。(4 分)

(二)上述電子與正子的粒子對產生過程只有當光子的頻率$\nu \geq \nu_0$時才會發生。ν_0稱為這個過程的底限頻率，求ν_0。(6 分)

三、 考慮邊長為 L 的立方體空腔。Planck's postulate: 頻率為 ν 的一個光子之平均能量為$\frac{h\nu}{\exp(h\nu/kT)-1}$ ，k 是波茲曼常數，T 是絕對溫度。(總分 15 分)

(一)空腔中頻率落在$[\nu,\ \nu + d\nu]$區間內，每單位體積的黑體輻射(電磁波)的能量為$\rho_T(\nu)d\nu$。由 Planck's postulate 求$\rho_T(\nu)$。(5 分)

(接下頁)

(二)求空腔中黑體輻射的能量密度。(5分)

(三)已知處於熱平衡的物體，其表面發射的黑體輻射強度遵守 Stefan Boltzmann law : $j = \sigma T^4$。由(一)、(二)小題的結果，求 Stefan Boltzmann constant σ。(5分)

四、考慮位能$V(x) = -V_0\{\delta(x + x_0) + \delta(x - x_0)\}$，$V_0 > 0$。對束縛態而言，能量$E < 0$。令$\kappa = \sqrt{-2mE}/\hbar$，這裡$m$為電子質量。(總分15分)

(一)若能量本徵態為偶函數，求此時κ滿足的方程式。(5分)

(二)若能量本徵態為奇函數，求此時κ滿足的方程式。(5分)

(三)V_0必須滿足什麼條件，波函數為奇函數的能量本徵態才會存在?(5分)

五、考慮一個初始時刻處於$4p$量子態的氫原子，若此氫原子在放出一個光子之後，躍遷到較低的能量本徵態。(總分15分)

(一)列出所有可能的末態。(6分)

(二)求這些躍遷所放出的光子之波長。(9分)

六、氫與氘原子為同位素，其原子核中都有一個質子，而氘的原子核另外還有一個中子。若已知氫分子H_2及氘分子D_2的解離能分別是 4.49 eV 及 4.57 eV。(總分15分)

(一)求這兩個分子振動能譜對應的等效彈力常數的數值，以 SI 單位表示之。(10分)

(二)承(一)小題。求 HD 分子的解離能，以 eV 表示之。(5分)

七、考慮自旋角動量$(\vec{S})$與軌道角動量$(\vec{L})$的合成：$\vec{J} = \vec{S} + \vec{L}$，$\vec{J}$稱為總角動量。(總分15分)

(一)若$s = 1/2$、$l = 1$，求所有可能之總角動量量子數(j, m_j)。(6分)

(二)呈上題，求所有可能之總角動量的量子態$|j, m_j\rangle$，以$|s, m_s\rangle \otimes |l, m_l\rangle$的線性組合表達之。(9分)

（試題隨試卷繳回）

教育部 111 年公費留學考試試題 112

科目：微積分　　　　　　　　　　　（全二頁，第一頁）

※以英文或中文作答皆可，評分基準相同，需寫出演算過程。

1. (10 points) Determine whether the statement is true or false. If it is true, prove it. If it is false, give a counterexample to show why it is false.
   (a) (5 points) Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ be continuous and $g:\mathbb{R}\longrightarrow\mathbb{R}$ be bounded. Then fg is continuous on $\mathbb{R}$.
   (b) (5 points) Let $f:\mathbb{R}^2\longrightarrow\mathbb{R}$ be a function of two variables. If the first-order partial derivatives f_x, f_y of f exist on $\mathbb{R}^2$, then f is differentiable on $\mathbb{R}^2$.

2. (15 points) Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ be an increasing function such that
$$f(x+y)=f(x)+f(y), \forall\ x,y\in\mathbb{R}.$$
   (a) (10 points) Show that f is a continuous function.
   (b) (5 points) Evaluate $\sum_{k=0}^{\infty} f((\frac{1}{\sqrt{3}})^k)$.

3. (15 points) The Mean Value Theorem states that : If f is continuous on a closed, bounded interval $[a,b]$ and differentiable on (a,b), then there exists $c\in(a,b)$ such that
$$f(b)-f(a)=f'(\mathrm{c})(\mathrm{b}-\mathrm{a}).$$
   Please give a rigorous proof of the Mean Value Theorem.

4. (15 points) Let $m\geq 2$ be an integer and $f(x)=x^{-m}$. Evaluate $\int_2^3 f(x)dx$ by taking the limit of the Riemann sums.

5. (15 points) Let a,b,c be real numbers and $\{x_n\}_{n\in\mathbb{N}}$ be a sequence of real numbers that satisfies the recursive relation
$$x_{n+3}=ax_{n+2}+bx_{n+1}+cx_n, \forall\ n\in\mathbb{N}$$
   with x_1,x_2 and x_3 are given.
   (a) (10 points) Suppose that the cubic equation $t^3-at^2-bt-c=0$ has three distinct real roots α,β and γ that are all nonzero. Show that the recursive sequence $\{x_n\}_{n\in\mathbb{N}}$ has the formula given by

(接下頁)

$$x_n = D\alpha^n + E\beta^n + F\gamma^n, \forall\ n \in \mathbb{N},$$

for some constants D, E and F. (You don't need to write down the constants D, E and F explicitly.)

(b) (5 points) Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence of real numbers that satisfies the recursive relation

$$x_{n+3} = 2x_{n+2} + 5x_{n+1} - 6x_n, \forall\ n \in \mathbb{N}$$

with $x_1 = 9$, $x_2 = 15$ and $x_3 = 63$. Determine whether the series $\sum_{n=1}^{\infty}\frac{1}{x_n}$ converges or diverges.

6. (20 points) Let $f(x) = \ln(1+x), x \in (-1,1)$.

(a) (5 points) Express $f(x)$ in the form of $f(x) = P_n(x) + R_n(x)$, where $P_n(x)$ is the *n-th* Taylor polynomial of f in powers of x and $R_n(x)$ is the corresponding remainder term.

(b) (10 points) Show that $\lim_{n\to\infty} R_n(x) = 0$ and then expand $f(x)$ as a Taylor series in powers of x.

(c) (5 points) Integrate the series to evaluate the sum

$$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k(k+1)}\cdot\frac{1}{2^{k+1}}.$$

7. (10 points) Let $D = \{(x,y) \in \mathbb{R}^2 | x^2 + y^2 \le 1\}$ be the unit disk and $f: D \to \mathbb{R}$ be continuous that satisfies

$$f\left(\frac{\sqrt{2}}{2}(x-y), \frac{\sqrt{2}}{2}(x+y)\right) = -f(x,y),\ \forall\ (x,y) \in D.$$

Evaluate the double integral $\iint_D f(x,y)\,dA$.

教育部 111 年公費留學考試試題 113

科目：線性代數 **(全一頁)**

※以中文或英文作答均可，評分基準相同。

Problem 1 Let $A=\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$ and $b=\begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$.

(i) (10 pts) Show that there does not exist a vector $x \in \Re^3$ that satisfies $Ax = b$.

(ii) (10 pts) Find a vector $y \in \Re^3$ such that $\| Ay - b \| \le \| Ax - b \|$ for all $x \in \Re^3$.

Problem 2 (15 pts) Let $M_2(\Re)$ be the collection of all 2×2 real matrices. Define $T: M_2(\Re) \to M_2(\Re)$ by $T(A) = CA - A^T$, where $C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Show that T is linear and find $\det(T)$.

Problem 3 (20 pts) Let U be an $n \times n$ complex matrix. Suppose U is unitary, that is $U^* U = I_n$. Show that $|tr(U)| \le n$. What can you say about U if $|tr(U)| = n$?

Problem 4 (15 pts) Let V and W be two finite dimensional vector spaces and $T: V \to W$ be a linear transformation. Show that T is one-to-one if and only if there exists a linear transformation $S: W \to V$ such that $ST = I_V$, where I_V is the identity map on V.

Problem 5 (15 pts) Let V be an n-dimensional vector space and $T: V \to V$ be linear. Suppose $v_1, v_2, \ldots, v_n$ are eigenvectors of T corresponding to distinct eigenvalues respectively. Let $x = v_1 + v_2 + \ldots + v_n$. Show that $\{x, T(x), \ldots, T^{n-1}(x)\}$ is a basis for V.

Problem 6 (15 pts) Let V be a finite dimensional vector space with more than one element and $T: V \to V$ be linear. Let $\ker(T)$ denote the kernel of T and $\operatorname{Im}(T)$ denote the image of T. Suppose $\ker(T) = \operatorname{Im}(T)$. Show that T is not diagonalizable.

(試題隨試卷繳回)

教育部 111 年公費留學考試試題 114

科目：數理統計與機率 （全二頁，第一頁）

※以中文或英文作答均可，評分基準相同。

※Justify your answers and provide detailed derivations for all the questions.

1. (21%) The random variable Y is called a Log-normal(μ, σ^2) distribution if $X = \log(Y)$ follows a Normal(μ, σ^2) distribution with mean μ and variance σ^2. Recall that the Normal(μ, σ^2) distribution has the density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \ -\infty < x < \infty.$$

   (a) (7%) Find the density function of *Y*.

   (b) (7%) Find $E(Y)$, the expectation of *Y*.

   (c) (7%) Find the median of *Y*.

2. (21%) Suppose that *X* has a Normal(μ, σ^2) distribution and that $Y = X + Z$, where *Z* is independent of *X* and has a Normal(θ, τ^2) distribution.

   (a) (7%) Find the marginal density function of *Y*.

   (b) (7%) Find the conditional density function of *Y* given *X*=*x*.

   (c) (7%) Find the conditional density function of *X* given *Y*=y.

3. (21%) Let $X_1, X_2, \dots, X_n$ be a sample from the density

$$f(x) = \exp(\theta - x), \ x \geq \theta, \ -\infty < \theta < \infty,$$

   where θ is the unknown parameter .

   (a) (7%) Find a sufficient statistic for θ based on the sample $X_1, X_2, \dots, X_n$.

   (b) (7%) Find $E(X)$ and the method of moment estimate of θ based on $E(X)$.

   (c) (7%) Find the maximum likelihood estimate for θ.

（接下頁）

4. (16%)Let (X_i, Y_i), $i = 1, \ldots, n$, be observations of X and Y. Assume that $Y_i = \alpha + \beta X_i + \varepsilon_i$, ε_i is independent of X_i and $E(\varepsilon_i) = 0$ for $i = 1, \ldots, n$.
   (a) (8%) Find the least squares estimate of (α, β).
   (b) (8%) Is the least squares estimate of (α, β) unbiased? Justify your answer.

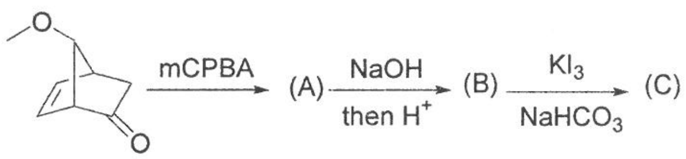
5. (21%)In the statistical hypothesis test regarding the mean μ of a normal distribution population with known variance σ^2, that is, the test $H_0: \mu = 0$ against $H_1: \mu > 0$, we take a random sample $X_1, X_2, \ldots, X_{25}$ from that population, and use for our test statistic $Y = \sum_{i=1}^{25} X_i / 25$. The rejection region is defined by $Y > c$ with $c > 0$. Let $\Phi(z) = \Pr(Z \leq z)$ be the cumulative distribution function (at a real number value z) of standard normal random variable Z.
   (a) (7%) Find the distribution or density of Y under the null hypothesis H_0.
   (b) (7%) Express the size (the type I error probability) of the test in terms of the standard normal distribution function Φ.
   (c) (7%) Express the power of this test at $\mu = c$ $(c > 0)$ in terms of Φ.

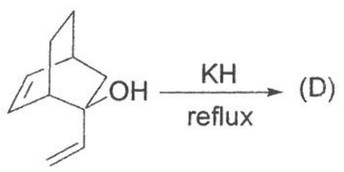
教育部111年公費留學考試試題 115

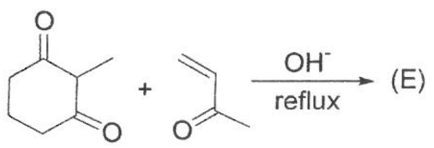
科目：有機化學 （全三頁，第一頁）

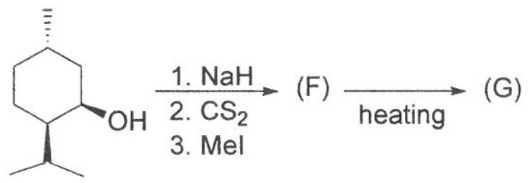
※以中文或英文作答均可，評分基準相同。

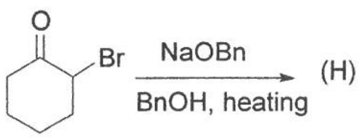
1. (40%, 4% each) Predict the product of the following reactions, show the stereochemistry of the product carefully.

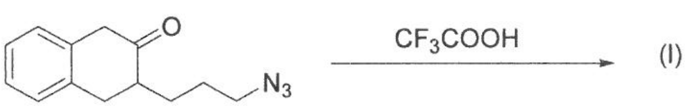


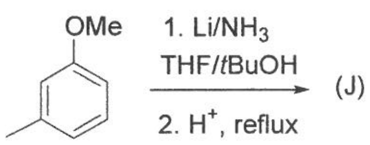








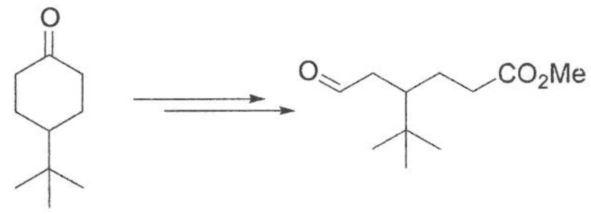




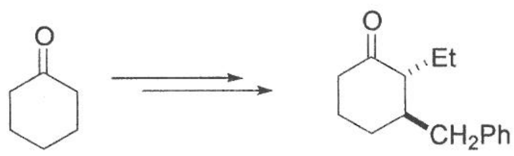
2. (4%) Arrange the basicity of the following substances: $CH_3CH_2C{\equiv}C^-$, $CH_3CH_2CH_2CH_2S^-$, $CH_3CH_2CH_2CO_2^-$, $CH_3CH_2CH_2CH_2O^-$, $CH_3CH_2CH_2CH_2NH^-$

3. (12%) Propose a synthesis from the given starting material to the product.

(a)



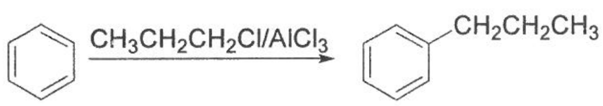
(b)



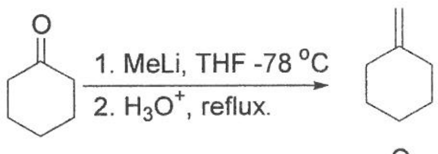
4. (18%, 6% for each) The following reactions can NOT work practically. Please explain why the reaction fails, (2%), and propose a synthesis to obtain the desired product (4%).

(這三個反應式都無法得到如圖預期的產物，請說明為何不能得到預期產物(2%)，與提出改進方法以得到預期產物(4%))。

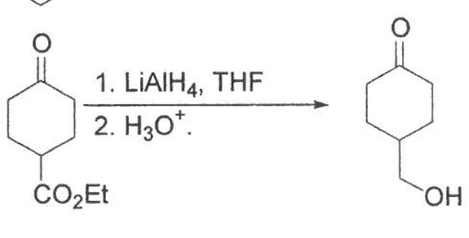
(a)



(b)

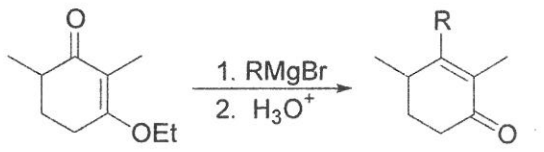


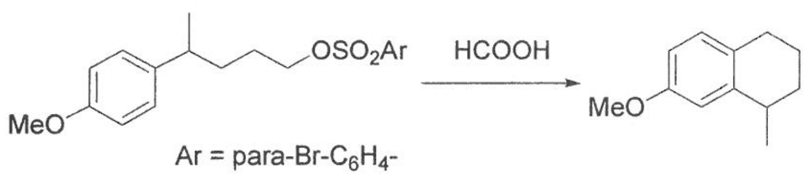
(c)

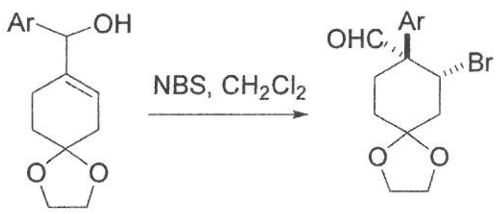


(接下頁)

5. (18%) Write a step-by-step mechanism for each of the following reactions.







6. (4%) Draw the [1]H and [13]C NMR spectra of the commercial available pure d-dichloromethane, CD_2Cl_2 (99% D), include the pattern and rough chemical shifts.

7. (4%) Determine the structure with the formula C_7H_8: [1]H-NMR: δ 2.0-2.2 (m, 2H), δ 3.5-3.7 (m, 2H), δ 6.7-6.9 (m, 4H); [13]C-NMR: δ 50.4 (d), δ 75.2 (t), δ 143.3 (d).

教育部111年公費留學考試試題 116

科目：無機化學 （全二頁，第一頁）

※可用中文或英文作答

一. 簡答題 (每題5分，60%)

1. Please indicate the point group of $[Co(ethylenediamine)_3]^{3+}$ in an octahedral geometry.

2. Please indicate the point group of $(\eta^5\text{-}C_5H_5)_2Fe$ in staggered form.

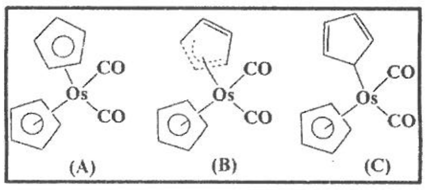
3. $[Cu(H_2O)_6]^{2+}$ has an elongated tetragonal geometry. What is the 3d orbital with the highest energy ?

4. What is the number of atoms in each unit cell of a body-centered cubic structure?

5. What are the lattice points of a body-centered cubic structure?

6. Use the Born-Harbor cycle to calculate the enthalpy of formation of LiF(s).
Use these data in the calculation: F_2 bond energy (dissociation energy) is 158 kJ/mol. The sublimation energy of Li(s) to Li(g) is 161 kJ/mol. The ionization energy of $Li_{(g)}$ to $Li^{+}_{(g)}$ is 520 kJ/mol. The electron affinity of $F_{(g)}$ to $F^{-}_{(g)}$ is -328 kJ/mol. The lattice enthalpy of $LiF_{(s)}$ is -950 kJ/mol.

Please answer the question of 7~9 related to compounds in the figure to the right.



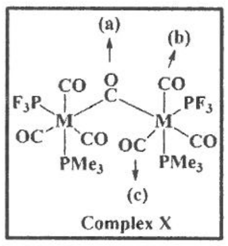
7. Determine the number of valence electrons for compound (A).

8. How many sets of peaks might appear in the ^{1}H NMR spectrum of compound (B) at very low temperature if hyperfine interaction is not considered?

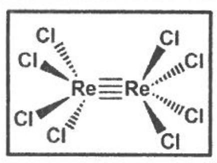
9. The ^{1}H NMR spectrum of compound (C) at very low temperature displays four sets of peaks. Please indicate the ratio of intensity for these four sets of peaks.

10. Please predict the base strength in the gas phase of a) $NHMe_2$, b) NH_2Me, c) NMe_3 and d) NH_3 in increasing order.

11. Complex X is shown in the figure to the right. Please predict the order of the bond distances for different CO groups (a, b, and c).



12. Please sketch the δ bonding interactions between two metal d orbitals in $[Re_2Cl_8]^{2-}$ as shown in the figure to the right.



二. 問答題 (40%)

(I) Please answer the equations for a $3d_{x2-y2}$ orbital whose angular wave function is shown to the right. (10%)

$$Y = \frac{1}{4}\sqrt{\frac{15}{\pi}}\frac{(x^2-y^2)}{r^2}$$

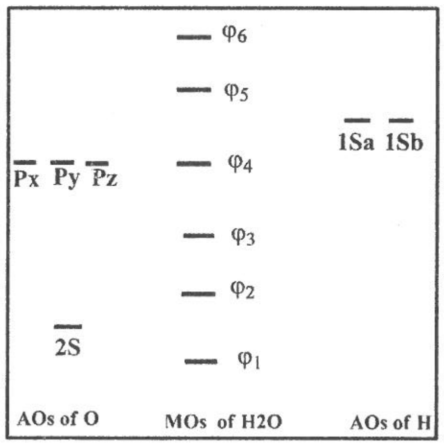
1. Please sketch d_{x2-y2} orbital and indicate angular nodal surfaces of this orbital.
2. What are equations for angular nodal surfaces of this orbital?
3. How many radial nodes does this orbital have?

教育部111年公費留學考試試題 116

科目：無機化學 **（全二頁，第二頁）**

(II) Please answer the following questions regarding molecular orbitals of H_2O. The atomic orbitals of oxygen are 2s, $2P_x$, $2P_y$, $2P_z$. The atomic orbitals for two hydrogens are $1S_a$ and $1S_b$. Please note that the C_2 axis is chosen as the z axis and the xz plane as the plane of the molecule. (10%)

C_{2v}	E	C_2 (z)	σ_v(xz)	σ_v(yz)	linear, rotations	quadratic
A_1	1	1	1	1	z	x^2, y^2, z^2
A_2	1	1	-1	-1	R_z	xy
B_1	1	-1	1	-1	x, R_y	xz
B_2	1	-1	-1	1	y, R_x	yz



1. What is the wave function for φ1? (The coefficients on the atomic orbitals do not have to be considered here.)
2. What is the LUMO orbital?
3. What is the wave function for φ4?
4. What is the irreducible representation of the symmetry for φ4?
5. φ6 orbital mainly consists of $2P_x$, $1S_a$ and $1S_b$ atomic orbitals. Please sketch the molecular orbital of φ6?

(III) Please answer questions regarding Fe(III) complexes with a trigonal bipyramidal geometry. (10%)

1. Please draw the d-orbital splitting diagram
2. Please label d-orbital in the diagram
3. Please write down all possible spin states of iron centers while binding to different types of ligand.

(IV) The formation constants of Ag^+ with halide ligands are $Br^- > Cl^- > F^-$ while, in contrast, the formation constants of Cu^{2+} with halides are $F^- > Cl^- > Br^-$. Please explain the differences. (5%)

Formation Constants (log *K*)

	NH_3	F^-	Cl^-	Br^-
Ag^+	3.30	−0.17	3.08	4.30
Cu^{2+}	4.24	0.9	0.09	−0.07

(V) The isoelectronic ions VO_4^{3-}, CrO_4^{2-}, MnO_4^- all have intense charge-transfer transition bands. The wavelengths of these transitions increase in this series, with MnO_4^- having its charge-transfer absorption at the longest wavelength. Please explain this trend. (5%)

（試題隨試卷繳回）