教育部 114 年公費留學考試試題 105

科目：計算機程式 **（全三頁，第一頁）**

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※以中文或英文作答均可，評分基準相同。

1. (10%) Given the following program:

```
int main()
{
   int nums [ ] = { 0, 20, 0, 8, 29, 25, 0 };
   int size = sizeof ( nums ) / sizeof
              ( nums [0] );
   func ( nums, size );
   for ( int i = 0; i < size; i++ )
      printf ( "%d ", nums [i] );
   return 0;
}
```

```
void func ( int* nums, int numsSize )
{
      int ans [ numsSize ];
      int j = 0;
      for ( int i = 0; i < numsSize; i++ )
      {
            if ( nums [i] != 0 )
            {
                  ans [j] = nums [i];
                  j++;
            }
      }
      while ( j < numsSize )
      {
         ans [j] = 0;
         j++;
      }
      for ( int i = 0; i < numsSize; i++ )
            nums [i] = ans [i];
}
```

(1) What are printed? (5%)

(2) Explain how you would evaluate the time complexity and space complexity of this code. (5%)

2. (10%) Show that the following statements are true or false.

(1) $2 + 4 + 8 + \ldots + 2^n = O(n^n)$ (5%)

(2) $5^n = O(4^n)$ (5%)

3. (10%) For any nonempty binary tree T, suppose that n_0 is the number of leaf nodes and n_2 the number of nodes of degree 2, prove $n_0 = n_2 + 1$.

（接下頁）

4. (10%) Write a program that reads an integer n and prints all odd squares in the inclusive range [1, n], in increasing order. Print the numbers separated by a single space.
   Example
   Input: 200
   Output:1 9 25 49 81 121 169

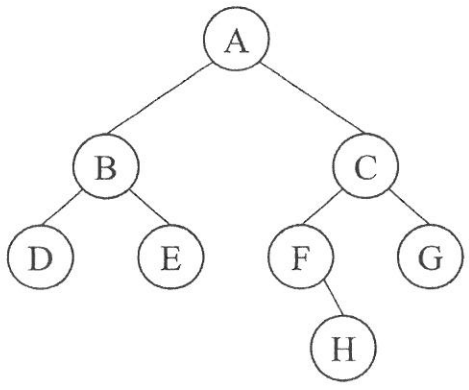
5. (10%) According to Kruskal's method for finding the minimum cost spanning tree, explain its idea, procedure, and time complexity.

6. (20%) Answer the following questions.

   (1) Convert the following infix expression into its prefix form: (5%)

   (A+B)*(C−D)/E

   (2) Given the following tree, write its postfix traversal. (5%)



   (3) Insert the numbers 15, 7, 20, 3, 10, 25, 2, 5 into an initially empty binary min heap. Draw the final heap. (5%)

   (4) Remove the smallest value from the heap obtained in (3) and reconstruct the min heap. Show the resulting tree. (5%)

7. (10%) Insert 30, 20, 40, 10, 25, 35, 50, 5 into an initially empty AVL tree (in this order) and draw the final tree.

8. (10%) What is the output when calling Test(3)?

```
String Test(int n) {
    String s = Test(n-2) + n + Test(n-1) + n;
    if (n <= 0) return "";
    return s;
}
```

（接下頁）

9. (10%) Using the Ackermann function defined below, compute the exact values of $A(2, 3)$ and $A(3, 2)$. Show your steps or justify your answers using known closed forms for fixed m. Assume $m, n > 0$.

$$A(m,n) = \begin{cases} n+1 & if\ m = 0 \\ A(m-1,1) & if\ m > 0\ and\ n = 0 \\ A\big(m-1, A(m,n-1)\big) & if\ m > 0\ and\ n > 0 \end{cases}$$

教育部 114 年公費留學考試試題 106

科目：線性代數 （全二頁，第一頁）

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※以中文或英文作答均可，評分基準相同。

1. (15%) Answer two questions regarding a linear system $Ax = b$, given an $n \times n$ matrix and an n-dimensional vector b.

   (a) Suppose we can find a solution to $Ax = b$, can we conclude that A is invertible? Explain your answer. (5%)

   (b) If b is the average of all the columns in A, what is the solution to $Ax = b$? (10%)

2. Write down the column rank and row rank of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. (10%)

3. (15%) Let us consider the following vectors:

$$\mathbf{t} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

   Suppose we also have $\mathbf{At} = \mathbf{Au} = \mathbf{Av} = \mathbf{0}$ and $\mathbf{Aw} = 2\mathbf{w}$ for a 4×4 matrix $\mathbf{A}$.

   Answer the following two questions.

   (a) Are $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$ linearly independent? (5%)

   (b) What could be the rank and nullity of $\mathbf{A}$? (10%)

4. Let $A = \begin{bmatrix} \sqrt{2}\cos\left(\frac{\pi}{6}\right) & -\sqrt{2}\sin\left(\frac{\pi}{6}\right) \\ \sqrt{2}\sin\left(\frac{\pi}{6}\right) & \sqrt{2}\cos\left(\frac{\pi}{6}\right) \end{bmatrix}$, find the polynomial p where we have $p(A) = \mathbf{0}$, with $\mathbf{0}$ denoting a 2×2 zero matrix. (10%)

（接下頁）

5. (20%) Answer two questions as follows.
   (a) If $\det(A) = -3$, calculate $\det(A^2)$ (10%)
   (b) If 0 is an eigenvalue of A, calculate $\det(AB)$. (10%)
   (Explain your reason, especially if you think it is not possible to find the answer.)

6. (20%) The Fibonacci sequence is defined as $a_0 = 0$, $a_1 = 1$ and $a_{n+2} = a_n + a_{n+1}$. Answer two questions to find the closed-form solution of the nth-term in the sequence.
   (a) Use a matrix form to write down the relation of consecutive terms in the sequence. That is, $\begin{bmatrix} a_{n+2} \\ a_{n+1} \end{bmatrix} = \mathbf{A}\begin{bmatrix} a_{n+1} \\ a_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} a_{n+1} \\ a_n \end{bmatrix}$. Find $\mathbf{A}$. (5%)
   (b) Apply diagonalization to the matrix $\mathbf{A}$ to find the general form of the nth-term in the Fibonacci sequence. (15%)

7. If U is any invertible $n \times n$ matrix, prove that $A = U^T U$ is positive definite. A matrix is called positive definite if $\mathbf{x}^T A\mathbf{x} > 0$, for every real-valued vector $\mathbf{x} \neq 0$. (10%)

教育部 114 年公費留學考試試題

科目：計算機概論　　（全一頁）

※以中文或英文作答均可，評分基準相同。

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

1. Sorting Algorithms (25 points)
   (a) Explain why comparison-based sorting cannot be faster than $O(nlogn)$ in the worst case. (10 points)
   (b) Suppose we want to sort an array of integers where each integer is in the range [1, n]. Which algorithm can achieve better than $O(nlogn)$ performance? Explain its time complexity. (10 points)
   (c) If QuickSort always picks the first element as pivot, give an example input that leads to worst-case time complexity. (5 points)

2. Trees and Graphs (25 points)
   (a) For a binary search tree (BST), explain the time complexity of search in the best and worst case. (10 points)
   (b) Given the following undirected graph: Vertices = {A, B, C, D, E}, Edges = {(A,B), (A,C), (B,D), (B,E), (C,D)}. Draw the adjacency matrix and the adjacency list. (5 points)
   (c) What are the traversal patterns from A to visit all other vertices by BFS and DFS respectively? (10 points)

3. Networking Protocols (25 points)
   (a) Explain the difference between TCP and UDP in terms of reliability and use cases. (10 points)
   (b) Describe the purpose of the 3-way handshake in TCP. (10 points)
   (c) If the round-trip time (RTT) is 100 ms, how long does it take to complete the TCP 3-way handshake before application data can be sent? (5 points)

4. Memory Management (25 points)
   (a) Define internal fragmentation and external fragmentation. Give one example of each. (10 points)
   (b) A system has 1 MB of RAM divided into fixed blocks of 4 KB. A program requests 4100 bytes. How much internal fragmentation occurs? (5 points)
   (c) A computer has: cache access time = 5 ns, main memory access time = 80 ns, cache hit rate = 92%. Derive the formula for Average Memory Access Time (AMAT) in terms of hit time, miss rate, and miss penalty and compute the AMAT with the given values. (10 points)

(試題隨試卷繳回)

教育部 114 年公費留學考試試題 108

科目：計算機網路 （全二頁，第一頁）

※以中文或英文作答均可，評分基準相同。

※可使用**工程**計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

Question 1: "Backbone Network"(25%)

a) Give *two* reasons why an Internet Service Provider (ISP) might select a route with a longer AS-PATH over a route with a shorter AS-PATH. (10%)

b) Suppose AS 1 has an AS-PATH of "1 2 3 4" to reach prefixes in AS 4. Suppose that an AS does not export BGP routes learned from one peer/provider to another peer/provider. Is AS 2 a customer, peer, or provider of AS 1? (5%)

c) Give *two* reasons why interdomain routing uses *path*-vector routing instead of *distance*-vector routing. (10%)

Question 2: Reliable Broadcast Channel (25%)

Consider a scenario in which a host, A, wants to simultaneously send messages to hosts B and C. A is connected to B and C via a broadcast channel --- a packet sent by A is carried by the channel to both B and C. Suppose that broadcast channel connecting A, B, and C can independently lose and corrupt messages (and so, for example, a message sent from A might be correctly received by B, but not by C). Design a stop-and-wait-like error-control protocol for reliably transferring a packet from A to B and C, such that A will not get new data from the upper layer until it knows that both B and C have correctly received the current packet. Give Finite-State-Machine (FSM) descriptions of A and B. (Hint: The FSM for C should be essentially the same as for B.)

（接下頁）

Question 3: "Quickies"(50%)

Answer each of the following questions *briefly, i.e., in at most a few sentences.*

a) List four factors that contribute to the end-to-end delay in a packet-switched network. Which of these are constant and which of these depend on the load in the network? (20%)

b) Suppose you would like to urgently deliver 50 terabytes data from Boston to Los Angeles. You have available a 1Gbps dedicated link for data transfer. Would you prefer to transmit the data via this link or instead use Fedex overnight delivery? Explain. (20%)

c) Please give 2 application protocols implemented (over) by TCP and 2 application protocols by UDP. (10%)

教育部 114 年公費留學考試試題 109

科目：資訊安全概論 **（全二頁，第一頁）**

※可使用**工程**計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※以中文或英文作答均可，評分基準相同。

一、(總分 20 分)請對以下問題進行論述：

(一) 資訊安全是保障「自由」還是「控制」？(4 分)

(二) 如果人類的行為永遠帶有漏洞，那麼「絕對安全」是否只是一種幻覺？(4 分)

(三) 黑客精神（hacker ethos）與資訊安全專業倫理之間是否存在根本衝突？(4 分)

(四) 安全投資的哲學基礎是「最大化存活機率」還是「最小化損失」？(4 分)

(五) AI 是否可能帶來「無人能理解的資訊安全決策」？這樣的安全還算「人類的安全」嗎？(4 分)

二、對稱式加密（AES）與非對稱式加密（RSA、ECC）的核心差異與適用場景？為什麼量子運算會對現有公鑰基礎建設（PKI）造成威脅？雜湊函數（Hash）如何用於密碼驗證與完整性檢查？(15 分)

三、(總分 15 分)請回答以下問題：

(一) HTTPS 如何確保傳輸安全？其中 TLS/SSL 的握手流程有什麼弱點？(5 分)

(二) 防火牆（Firewall）與入侵偵測系統（IDS）/入侵防禦系統（IPS）的主要差異？(5 分)

(三) VPN 與 Zero Trust（零信任架構）在企業環境中的比較？(5 分)

四、行政院國土安全辦公室 2025 年公布的「國家關鍵基礎設施安全防護指導綱領」中，關鍵技術設施領域的範圍包括那些領域？(15 分)

（接下頁）

教育部 114 年公費留學考試試題 109

科目：資訊安全概論 （全二頁，第二頁）

五、 SQL Injection、XSS、CSRF 這些常見 Web 攻擊的原理與防禦方法？Buffer Overflow 為什麼危險？現代作業系統有哪些緩解措施（如 ASLR、DEP）？為什麼「軟體更新」是安全中最有效卻最容易被忽視的技術？(15 分)

六、 組織為什麼需要資訊安全政策？資訊安全長（CISO）的職責與挑戰是什麼？如何將資訊安全納入企業治理，而不是只當作 IT 部門的事？企業如何在安全與營運效率之間取得平衡？(10 分)

七、 已知 RSA 選取質數 p=11、q=13，若選擇公鑰 e=7，請計算對應的私鑰 d。(10 分)

（試題隨試卷繳回）

教育部 114 年公費留學考試試題　110

科目：離散數學　　（全一頁）

※可使用工程計算機(限僅具備＋、－、×、÷、%　、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※以中文或英文作答均可，評分基準相同。

1. (20%) Consider the following logical expression:

$$\neg\,(\,(p \wedge q) \leftrightarrow (p \rightarrow (\neg q))\,)$$

Find an equivalent expression that is as short as possible.

2. (15%) Show that 120 divides the product of any 5 consecutive positive integers.

3. (15%) Let n be a positive integer. Show that any $n(n-1)+1$ integers must contain n numbers whose sum is divisible by n.

4. (15%) Let $\gcd(x, y)$ denote the greatest common divisor of x and y. Find the number of integers $k \in [1, 720]$ with $\gcd(k, 720) > 1$.

5. (20%) Find the coefficient of x^n in the following generating function:

$$[\,(1-2x)(1-3x)\,]^{-1}$$

6. (15%) Let n be a positive integer greater than or equal to 3. Find the number of subgraphs of an n-vertex complete graph K_n that is isomorphic to a cycle with n vertices.

（試題隨試卷繳回）

教育部 114 年公費留學考試試題

科目：半導體物理 （全四頁，第一頁）

※以中文或英文作答均可，評分基準相同。

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

1.(25%) A uniformly doped n-type silicon sample at T = 300 K has a donor concentration $N_D = 1.0 \times 10^{16}\ cm^{-3}$. Assume complete ionization, non-degenerate statistics, and intrinsic carrier concentration $n_i = 1.5 \times 10^{10}\ cm^{-3}$. An optical source is switched on at t = 0, producing a uniform excess-carrier generation rate $g' = 1.0 \times 10^{21}\ cm^{-3} \cdot s^{-1}$. The electric field is zero. Use $kT/q = 0.0259$ eV at 300 K.

(a) Under thermal equilibrium (for $t < 0$), determine the Fermi level E_F with respect to the intrinsic level E_i. (5%)

(b) In steady state under illumination, the measured maximum excess concentrations are $\Delta n_{ss} = \Delta p_{ss} = 2.0 \times 10^{14}\ cm^{-3}$. Determine the minority-carrier lifetime τ. (5%)

(c) Derive $\Delta n(t)$ and the excess recombination rate $R'(t)$ for $t \geq 0$, given the initial condition $\Delta n(0) = 0$. Then find the times at which Δn equals one-half of the steady-state value. (7%)

(d) At $t = 0.30\ \mu s$, estimate the electron and hole quasi-Fermi levels E_{Fn} and E_{Fp} relative to E_i. Compare with the equilibrium E_F. (8%)

2.(25%) Consider an abrupt p^+n junction under zero bias. The p-side is degenerately doped such that the Fermi level lies well inside the valence band, while on the n-side, the conduction-band edge is nearly aligned with the Fermi level far from the junction.

（接下頁）

(a) Sketch the energy-band diagram at equilibrium (zero bias). (5%)

(b) Sketch the current–voltage (I–V) characteristics under forward and reverse bias. Explain why it behaves in such characteristics clearly. (5%)

(c) These diodes are often used in high-frequency and microwave applications. Explain what property makes them suitable for such applications. (5%)

(d) Assume the junction is abrupt, with doping concentrations: $N_A = 5 \times 10^{19}\ cm^{-3}$ on the p^+ side, $N_D = 1 \times 10^{17}\ cm^{-3}$ on the n side. At T = 300 K, take $V_{bi} = 0.8\ V$ and semiconductor permittivity $\varepsilon_s = 11.7\,\varepsilon_0$ with $\varepsilon_0 = 8.854 \times 10^{-14}\ F/cm$.

(i) Which side of the junction does the depletion region extend more into? Explain.

(ii) Estimate the depletion width on each side under zero bias. (iii) Plot the electric-field distribution across the junction. Label the plot clearly. (10%)

3.(25%) Consider an ideal MOS capacitor with a p-type substrate and an n^+ polysilicon gate.

(a) Plot the low-frequency (LF) and high-frequency (HF) C–V from accumulation to inversion over the same range. Explain the differences between the two curves. Label C_{ox}, C_{min}, V_{FB}, and V_T. (5%)

(b) Assume a uniform positive fixed oxide charge Q_{ox}. Sketch how the HF C–V curve compares with the ideal HF C–V and indicate the direction of the flat-band shift ΔV_{FB}. Provide a short physical explanation. (5%)

（接下頁）

(c) Assume there are donor-like interface traps (D_{it}) located (i) near E_V, (ii) around midgap, and (iii) near E_C. Sketch three separate HF C–V curves on top of the ideal HF curve. Indicate and explain the differences resulting from the D_{it} in the curves. (6%)

(d) If we have an nMOS built on the same p-type substrate, please sketch and label on a semi-log $I_D - V_G$ plot at a small drain bias V_D. (i) the ideal transfer curve, (ii) the effect of positive Q_{ox}, and (iii) the effect of high D_{it} distributed near midgap. Compare the threshold-voltage shift and the subthreshold swing qualitatively. (9%)

4.(25%) A uniformly doped n^+-p^+-n bipolar transistor is one-dimensional and at T=300 K. Assume complete ionization, non-degenerate statistics, and low-level injection. Let the emitter–base (E–B) and base–collector (B–C) metallurgical junctions be abrupt. The base width is W_B. Assume diffusion and recombination occur only in the neutral base region. Let D_n and $L_n = \sqrt{D_n \tau_n}$ be the electron diffusion coefficient and diffusion length in the p^+ base.

(a) In thermal equilibrium, plot the energy-band diagram across emitter–base–collector and indicate the E_C, E_V, and the Fermi level E_F. (5%)

(b) In thermal equilibrium, plot the electric-field distribution across the device and indicate where the field is non-zero and which depletion region is wider. (5%)

（接下頁）

(c) In the forward-active operation, plot the energy-band diagram and add the quasi-Fermi levels E_{Fn} and E_{Fp}. (5%)

(d) Under steady forward-active operation, plot the minority-electron excess concentration $\Delta n(x)$ in the p^+ base for $0 \leq x \leq W_B$, where $x = 0$ is the E–B depletion edge and $x = W_B$ is the B–C depletion edge. Please plot $\Delta n(x)$ for three cases: $L_n / W_B = 0.5, 1, 5$, and state the short-base limit. Briefly comment on how L_n / W_B influences current gain and base transit time. (10%)

(試題隨試卷繳回)

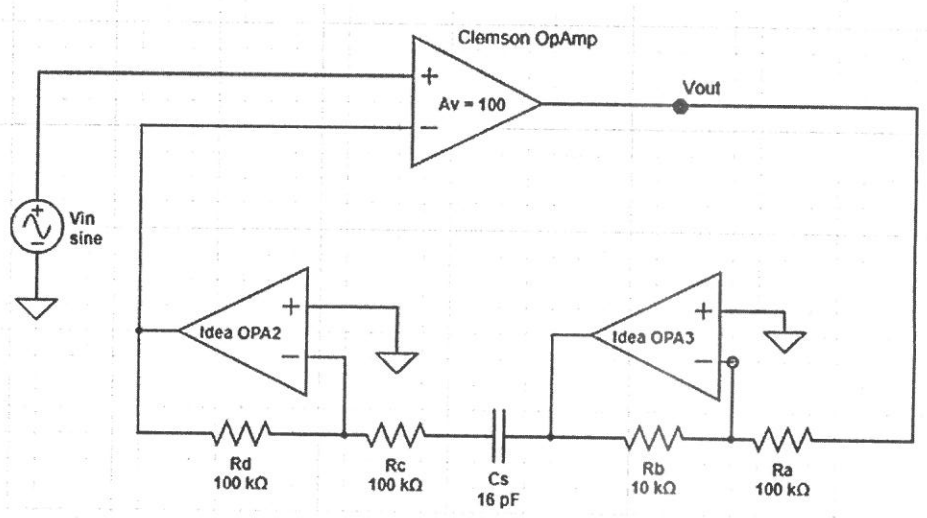
教育部 114 年公費留學考試試題 112

科目：電子學 （全二頁，第一頁）

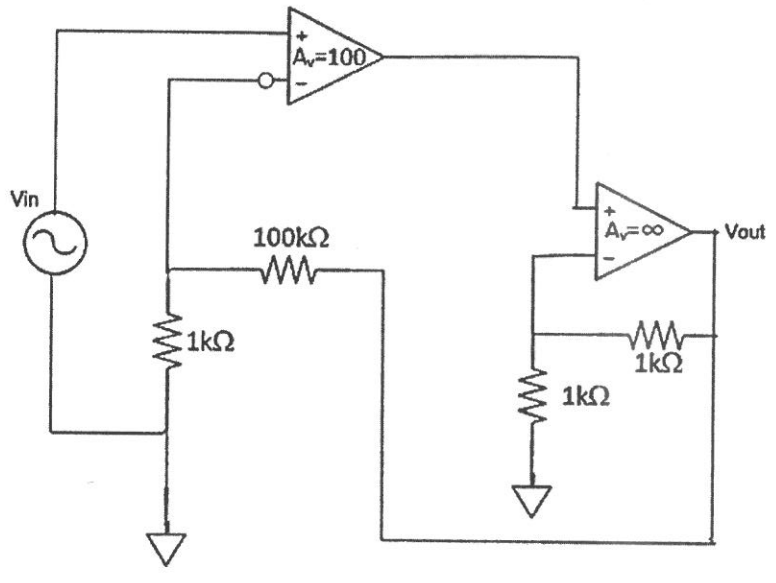
※以中文或英文作答均可，評分基準相同。

※可使用工程計算機(限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

1. (20 points) Sketch and Label the Bode plot showing the voltage gain in all flat regions and the break frequencies (gain in dB vs Log of Frequency). Here OPA2 and OPA3 are considered ideal in every way, but the Clemson OpAmp is ideal in all properties except its open-loop gain is only 100 V/V.



2. (20 points) There are two OpAmps., which are the OPA with an open-loop gain of 100 V/V and the OPA one is considered ideal listed in the following figure. Please determine the closed loop gain.



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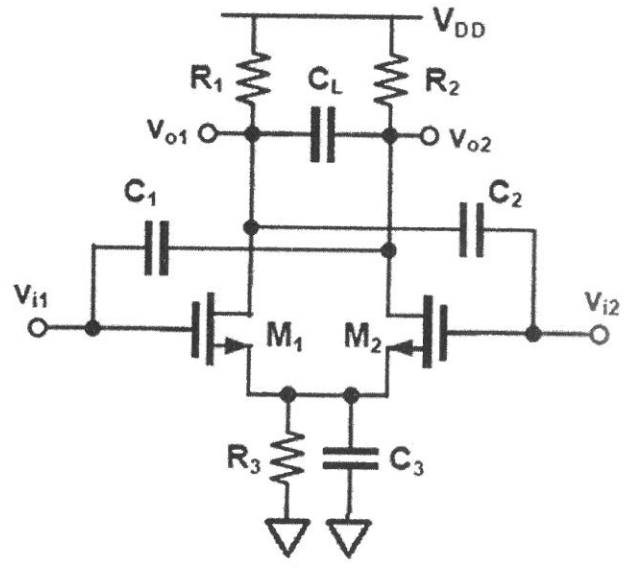
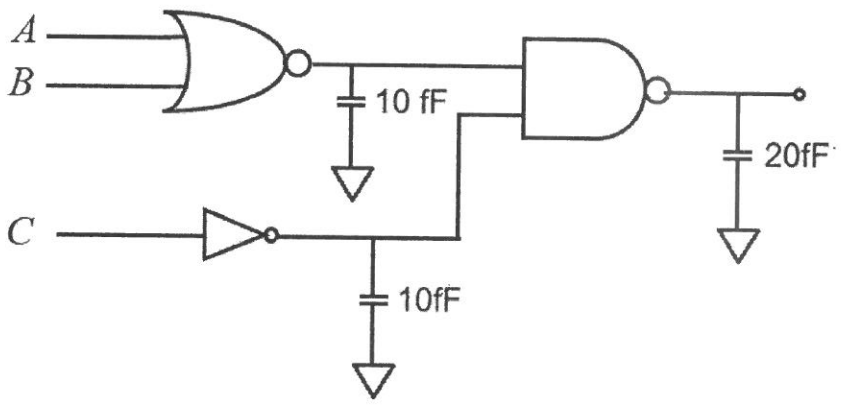
3. (40 points) A differential amplifier circuit is as shown in the following figure. The transistors have transconductance g_m equal to 1 mA/V, and parasitic capacitances C_{gs}=0.5pF, and C_{gd}=0.1pF. Ignore transistor output resistance. Circuit component values are designed with R_1=R_2=15kΩ, R_3=5kΩ, C_1=C_2=C_3=0.1pF, CL=5pF.
(a) Make a Bode plot showing the frequency response of the differential-mode voltage gain magnitude. Mark the values of the corner frequencies. (20 points)
(b) Make a Bode plot showing the frequency response of the common-mode voltage gain magnitude. Mark the values of the corner frequencies. (20 points)



4. (20 points) Find the dynamic power dissipation, P_{dyn}, for the following CMOS circuit assuming the clock is 1GHz, V_{DD}= 2.5V and the inputs have the following probability values that change on the rising edge of the clock. P(A=1)=0.5, P(B=1)=0.4, and P(C=1)=0.7.



（試題隨試卷繳回）