科目：普通物理　　　　　　　　　　　　（全二頁，第一頁）

※可使用一般計算機(限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-運算功能)

※以中文或英文作答均可，評分基準相同。

1. (20%) A string under a tension of 50.0 N is used to whirl a rock in a horizontal circle of radius 2.50 m at a speed of 20.4 m/s on a frictionless surface as shown in Figure 1. As the string is pulled in, the speed of the rock increases. When the string on the table is 1.00 m long and the speed of the rock is 51.0 m/s, the string breaks. What is the breaking strength, in newtons, of the string?
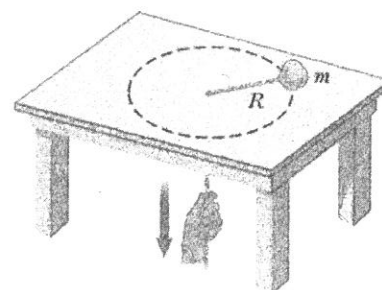
Figure 1

2. (20%) A pendulum of length L and mass M has a spring of force constant k connected to it at a distance h below its point of suspension (see Figure 2). Find the frequency of vibration of the system for small values of the amplitude (small θ). Assume the vertical suspension rod of length L is rigid, but ignore its mass.
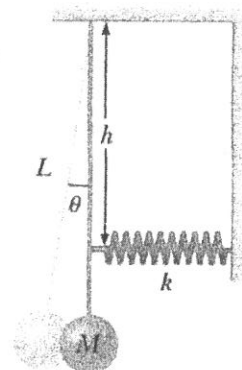
Figure 2

3. (20%) A uniformly charged rod of length L and total charge Q lies along the x axis of an x-y plane at the interval between $0 \leq x \leq L$.

   (a) Find the components of the electric field at the point P on the y axis which lies at a distance d from the origin of the x-y coordinate. (10%)

   (b) What are the approximate values of the field components when $d \gg L$? Explain why you would expect these results. (10%)

（接下頁）

4. (20%) A long, cylindrical conductor of radius R carries a current I as shown in Figure 3. The current density is J, however, is not uniform over the cross section of the conductor but rather is a function of the radius according to $J=br$, where b is a constant and r is the radius. Find an expression for the magnetic field magnitude B for the following two scenarios : (a) at a distance $r_1 < R$ and (b) at a distance $r_2 > R$, measured from the center of the conductor. (10% each)
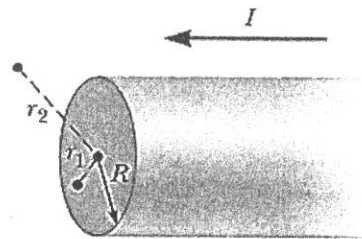

Figure 3

5. (20%) A conducting rod moves with a constant velocity in a direction perpendicular to a long, straight wire carrying a current I as shown in Figure 4. Show that the magnitude of the emf generated between the ends of the rod is
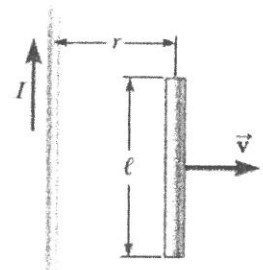
$$|\varepsilon| = \frac{\mu_0 vIL}{2\pi r}$$

Figure 4

教育部 113 年公費留學考試試題

科目：量子物理　　　　　　　　　　　　　　　（全二頁，第一頁）

※可使用**一般**計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-運算功能)

一、(總分 25 分)艾倫費斯特定理（Ehrenfest Theorem）說明，任何量子力學算符的期望值與該算符和物理系統中的漢彌爾頓量（Hamiltonian）之交換子的期望值之間存在特定關係，這其實是一個更普遍定理的特例：$\frac{d}{dt}\langle A\rangle = \frac{1}{i\hbar}\langle[A,H]\rangle + \langle\frac{\partial A}{\partial t}\rangle$。其中，$A$ 是量子力學中的算符，而 $\langle A\rangle$ 表示其期望值。需要注意的是，此普遍定理實際上並非源自艾倫費斯特，而是由海森堡（Heisenberg）所提出。假設 p 和 V 分別表示系統的動量和位能。

(一) 請證明 $\frac{d}{dt}\langle A\rangle = \frac{1}{i\hbar}\langle[A,H]\rangle + \langle\frac{\partial A}{\partial t}\rangle$。(15 分)

(二) 請證明 $\frac{d}{dt}\langle p\rangle = \langle-\frac{\partial V}{\partial x}\rangle$。此結果表明，期望值遵循古典力學規律。
(10 分)

二、(總分 25 分)簡諧振子的同調態。在簡諧振子的能量本徵態 $|n\rangle = \psi_n(x)$ 中，只有當 $n = 0$ 時，不確定性乘積可達到極限的最小值 $\hbar/2$。然而，特定的線性組合，稱為同調態，亦可使不確定性乘積最小化。此同調態滿足下降算符的本徵方程：$a_-|\alpha\rangle = \alpha|\alpha\rangle$。其中，本徵值 α 可以是任意的複數。提示：

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)\ ;\ p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)。$$

(一) 請計算在同調態 $|\alpha\rangle$ 中的期望值 $\langle x\rangle$、$\langle x^2\rangle$、$\langle p\rangle$ 和$\langle p^2\rangle$。(16 分)

(二) 請計算其位置和動量的不確性量 σ_x 和 σ_p，並證明$\sigma_x\sigma_p = \hbar/2$。
(9 分)

三、(總分 25 分)測量自旋角動量分量的期望值。設想一個自旋 1/2 粒子處於下列量子態：$\psi = \frac{1}{\sqrt{3}}\begin{pmatrix} 1+i \\ 1 \end{pmatrix}$。在量子力學中，自旋 1/2 粒子的自旋角動量分量（S_x、S_y、S_z）可以用包立矩陣來表示。具體地說，自旋角動量的分量與包立矩陣的關係如下：$S_x = \frac{\hbar}{2}\sigma_x$，$S_y = \frac{\hbar}{2}\sigma_y$，$S_z = \frac{\hbar}{2}\sigma_z$。其中，包立矩陣為：$\sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$、$\sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$、$\sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$。這些包立矩陣是自旋角動量分量的數學描述工具，用於計算自旋角動量的期望值。

(一) 當測量自旋角動量的 S_z 和 S_y 分量時，請計算獲得 $+\hbar/2$ 和 $-\hbar/2$ 的機率分別是多少？(15 分)

(二) 請計算自旋角動量分量 S_z 及 S_y 的期望值。(10 分)

四、(總分 25 分)兩個全同玻色子被置於一個無限位能井中。兩者之間的交互作用微弱，位能表達為：$V(x_1, x_2) = -aV_0\delta(x_1 - x_2)$。其中，$V_0$ 是能量尺度的常數，a 是位能井的寬度。

(一) 忽略兩粒子間的交互作用，請找出系統的基態和第一激發態，並給出對應的波函數和能量。(10 分)

(二) 使用一階微擾理論來處理兩粒子之間的微弱交互作用，估算這種交互作用對基態和第一激發態能量的修正效應。具體來說，利用未受擾的基態與激發態波函數，計算微擾位能對能量的影響。(15 分)

科目：微積分　　　　　　　　　　　　　　　（全二頁，第一頁）

※可使用一般計算機（限僅具備＋、－、×、÷、％ 、√、MR、MC、M+、M−運算功能）

※以英文或中文作答皆可，評分基準相同，需計算過程。

1. (15 points) Use the fact $\int_0^\infty \sin x/x \ dx = \pi/2$ to show that
   (a) (7 points)
   $$\int_0^\infty \frac{\sin x \cos x}{x} dx = \frac{\pi}{4}.$$

   (b) (8 points) Obtain
   $$\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}.$$

2. (7 points) Let $[\cdot]$ be a gauss symbol. Use a Riemann-Stieltjes integral to derive the following formula:
   $$\sum_{k=1}^n \frac{1}{k^s} = \frac{1}{n^{s-1}} + s \int_1^n \frac{[x]}{x^{s+1}} dx \quad \text{for any } s \neq 1.$$

3. (8 points) Define
   $$G(y) = \int_0^\infty e^{-xy} \frac{\sin x}{x} dx \ \text{ for } y > 0.$$
   Show that $G'(y) = -1/(1 + y^2)$.

4. (15 points) Find a general solution of the differential equation:
   $(x - 2y)dx + (2x + y)dy = 0.$

科目：微積分　　　　　　　　　　　　　　　（全二頁，第二頁）

5. (15 points) Let

$$f(x) = \sum_{n=1}^{\infty} n^2 x^n$$

for any $|x| < 1$. Calculate the value $f(1/2)$.

6. (10 points) Find a series of polynomials $f(x) = \sum_{n=1}^{\infty} p_n(x)$ which satisfies that $f(x)$ is convergent for $|x| < 1$ but $\lim_{x \to 1} f(x) \neq \sum_{n=1}^{\infty} p_n(1)$.

7. (15 points) Let $a_0 = 1$ and $a_n = a_{n-1} - a_{n-1}^3/6$. Calculate $\lim_{n \to \infty} \sqrt{n}\, a_n$.

8. (15 points) Let $f(x)$ be a continuous function on $[0,1]$, $f(0) = 0$, and $f(1) = 1$. Assume that f' and f'' exist and $f'' > 0$ on $(0,1)$. Show that $f(x) < x$ for any $x \in (0,1)$.

科目：線性代數　　　　　　　　　　　　　　（全二頁，第一頁）

※可使用一般計算機（限僅具備＋、－、×、÷、％ 、√、MR、MC、M+、M–運算功能）

※以中文或英文作答均可，評分基準相同。

1. (20 pts) Let V be the complex vector space of functions spanned by the basis $\alpha = \{1, x, \sin(x), \cos(x)\}$, and let T be the differential operator defined on V by $T(f(x)) = f'(x)$.

   (1) Find the matrix representation of the operator T with respect to the basis α. (10 pts)

   (2) Find the Jordan canonical form of the operator T. (10 pts)

2. (20 pts) Let T be the rotation around the unit vector $(\frac{1}{3}, \frac{2}{3}, \frac{-2}{3})$ by $30°$ counterclockwise in the real space $\mathbb{R}^3$.

   (1) Find the matrix representation A of the rotation T with respect to the standard basis $\{(1,0,0), (0,1,0), (0,0,1)\}$ of $\mathbb{R}^3$. (10 pts)

   (2) Find all 3×3 real matrices that commute with A. (10 pts)

3. (20 pts) Consider the following sequence defined by $a_1 = 3, a_2 = 4$, and $a_n = a_{n-1} + 6\,a_{n-2}$ for all $n \geq 3$.

   (1) Find a matrix A such that: $\begin{pmatrix} a_n \\ a_{n-1} \end{pmatrix} = A \begin{pmatrix} a_{n-1} \\ a_{n-2} \end{pmatrix}$ for all $n \geq 3$. (5 pts)

   (2) Find the general formula for a_n. (15 pts)

4. (20 pts) Let A and B be two commuting $n \times n$ complex matrices acting on the complex vector space $\mathbb{C}^n$.

   (1) Prove or disprove that every A-invariant subspace is also B-invariant. (5 pts)

   (2) Prove or disprove that every eigenspace of A subspace is also B-invariant. (5 pts)

   (3) Show that if A and B are both diagonalizable, then they can be simultaneously diagonalizable. (10 pts)

5. (20 pts) Let A be an $n \times n$ positive-definite matrix.

    (1) Write down the definition of a positive definite matrix. Do all diagonal entries of A have to be positive? Justify your answer. (10 pts)

    (2) Show that there exists a unique positive-definite matrix B such that $B^2 = A$. (10 pts)

科目：有機化學　　　　　　　　　　　　　　　（全六頁，第一頁）

※可使用一般計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M−運算功能)

※以中文或英文作答均可，評分基準相同。

**1.** (30%) Single choice for each of the following questions.

(1)　Which of the following correctly lists the conformations of cyclohexane in order of increasing energy? (2%)
(A) chair < boat < half-chair < twist-boat
(B) chair < twist-boat < half-chair < boat
(C) half-chair < boat < twist-boat < chair
(D) half-chair < twist-boat < boat < chair
(E) chair < twist-boat < boat < half-chair

(2)　When methanol (CH_3OH) acts as a base, its conjugate acid is _____? (2%)
(A) $^-CH_2OH$
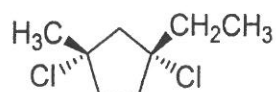(B) CH_3O^-
(C) CH_4OH
(D) $CH_3OH_2^+$
(E) CH_4O^+

(3)　In the lowest energy conformation of the compound shown below, how many alkyl substituents are axial? (2%)
(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
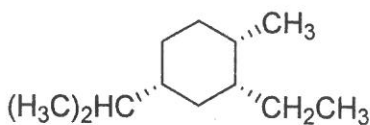
(4)　Which of the following compounds is chiral? (2%)
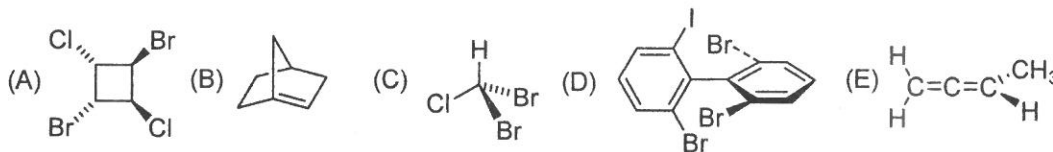
(5)　What is the IUPAC name of the compound shown below? (2%)

(A) (1S,3S)-1,3-dichloro-1-ethyl-3-methylcyclopentane
(B) (1R,3S)-1,3-dichloro-1-methyl-3-ethylcyclopentane
(C) (1R,4S)-1,4-dichloro-1-ethyl-4-methylcyclopentane
(D) (1S,3R)-1,3-dichloro-3-ethyl-1-methylcyclopentane
(E) (1R,3S)-1,3-dichloro-1-ethyl-3- methyl- cyclopentane

(6) Which of the following solvents could be described as polar aprotic? (2%)
(A) ethanol
(B) methylene chloride
(C) acetonitrile
(D) THF
(E) diethyl ether

(7) Rank the following species in order of decreasing acid-dissociation constant: (I) methanol, (II) ethanol, (III) 2-chloroethanol, (IV) *t*-butyl alcohol, and (V) phenol. (2%)
(A) V > III > I > II > IV
(B) V > IV > III > II > I
(C) III > V > I > IV > II
(D) V > I > III > II > IV
(E) III > IV > V > II > I

(8) Which of the following compounds is suitable solvent for Grignard reactions? (2%)
(A) $(CH_3)_2O$
(B) benzene
(C) methanol
(D) acetone
(E) chloroform

(9) Which of the following compounds can be reduced by $LiAlH_4$ to form <u>primary</u> alcohol?　(I) Aldehyde, (II) ketone, (III) alkene, (IV) acid, and (V) acetylene. (2%)
(A) I only
(B) I and II
(C) I and IV
(D) III and IV
(E) IV only

(10) Which of the following statements about the heats of combustion of acetylene, ethene, and ethane is <u>incorrect</u>? (2%)
(A) Acetylene is the best high-temperature flame
(B) Acetylene release the most heat per mole of products formed
(C) Acetylene produced the most heat per mole of oxygen consumed
(D) Ethane produced the most heat per mole of oxygen consumed
(E) The oxyacetylene flame reaches temperatures as high as 3500 °C

科目：有機化學　　　　　　　　　　　　　　　（全六頁，第三頁）

(11) Which of the following methods will convert propene to 2-propanol?
(I) H_2O/H_2SO_4, (II) $BH_3 \cdot THF$; H_2O_2/HO^-, (III) $Hg(OAC)_2$, H_2O; $NaBH_4$, (IV) H_2/Pt, (V) Br_2/H_2O. (2%)
(A) I only
(B) II only
(C) I and III
(D) III and IV
(E) V only

(12) Which of the following statements about the basicity and acidity of typical organic compounds is <u>incorrect</u>? (2%)
(A) Acetylene is more acidic than alcohol
(B) Ammonia (NH_3) is more acidic than ethylene
(C) Acetylene is more acidic than ammonia
(D) Acetylene can be deprotonated by amide ($^-NH_2$) ion
(E) Acetylene can be deprotonated by alkyl lithium

(13) Rank each of the following compounds in order of increasing heat of hydrogenation.　(I) Penta-1,2-diene, (II) pent-2-yne, (III) penta-1,4-diene, (IV) *trans*-penta-1,3-diene, and (V) *trans*-hexa-1,4-diene. (2%)
(A) V < III < IV < II < I
(B) V < IV < C < II < I
(C) IV < V < II < III < I
(D) I < II < III < V < IV
(E) IV < V < III < II < I

(14) Which of the following aromatic compounds has the lowest resonance energy? (2%)
(A) Benzene
(B) Pyrrole
(C) Furan
(D) Thiophene
(E) Pyridine

(15) According to Hückel's rule which of the following compounds is aromatic? cyclobutadiene (I), cyclopentadienyl anion (II), pyrrole (III), imidazole (IV), and cyclooctatetraene (V). (2%)
(A) II only
(B) II, III, and IV
(C) II and III only
(D) I and IV only
(E) All of the above

（接下頁）

2. The IR and MS of an unknown compound **A** are shown below. Use everything you know about IR and MS, plus reasoning and intuition, to determine a likely structure of **A**. Then show how your proposed structure is consistent with all these spectra. Namely, you need to explain the major peaks in both IR and MS. (10%)
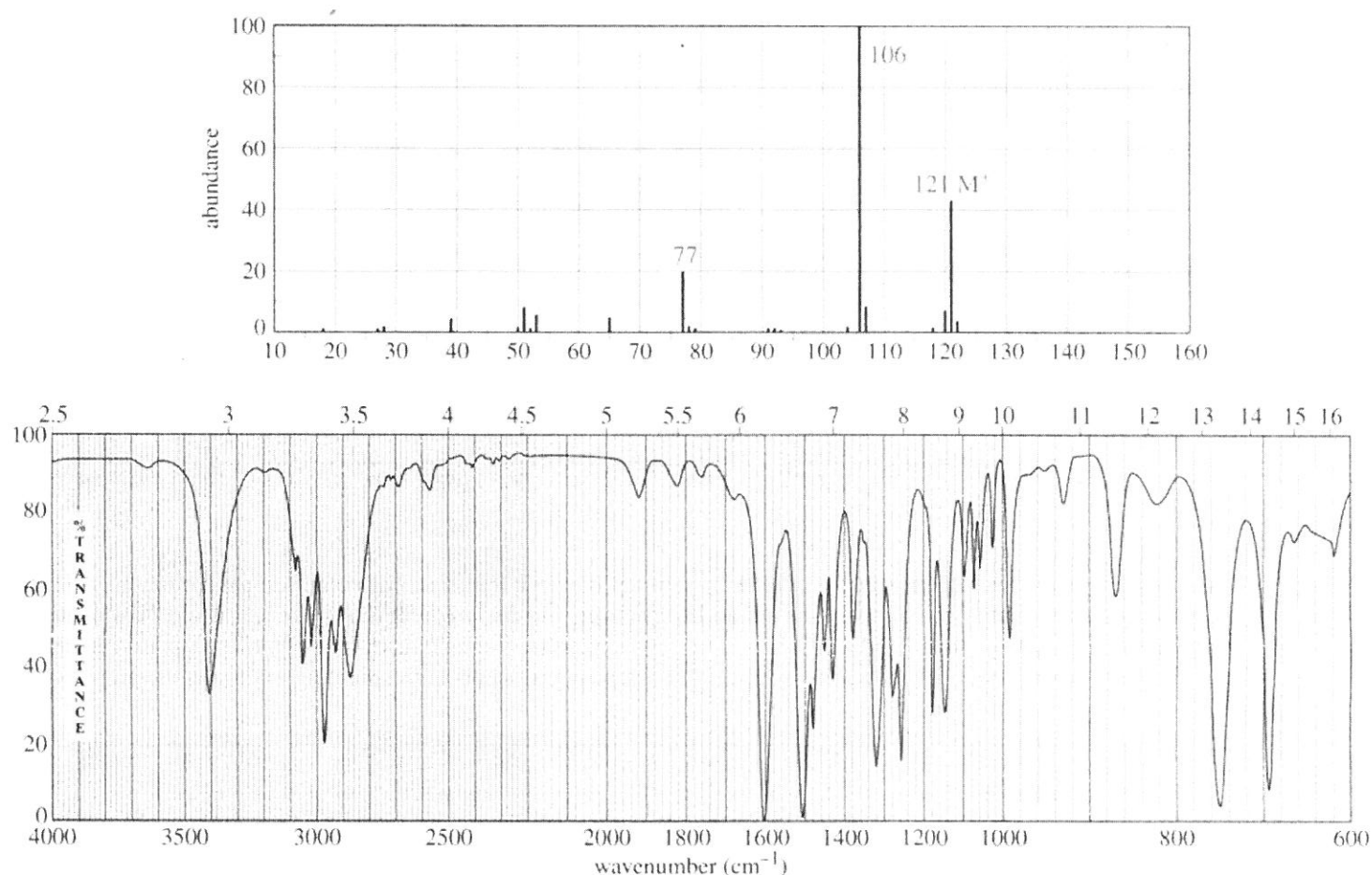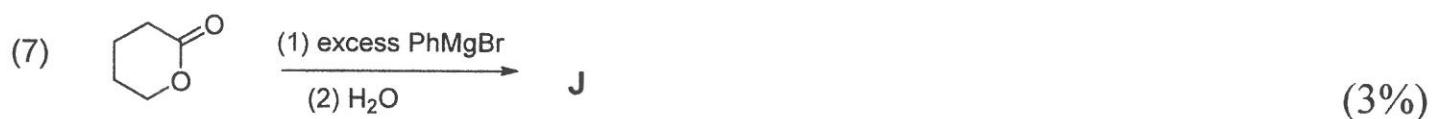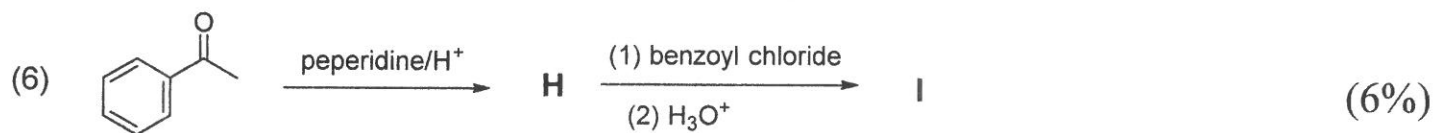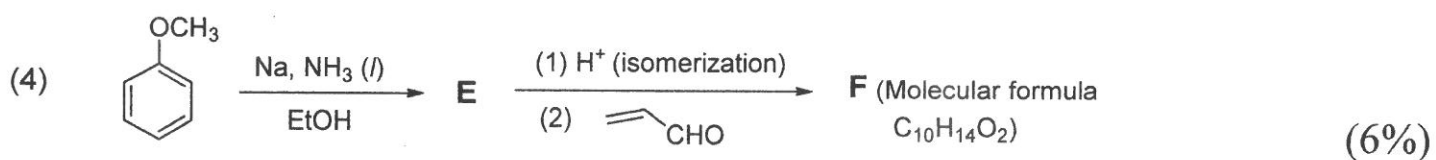

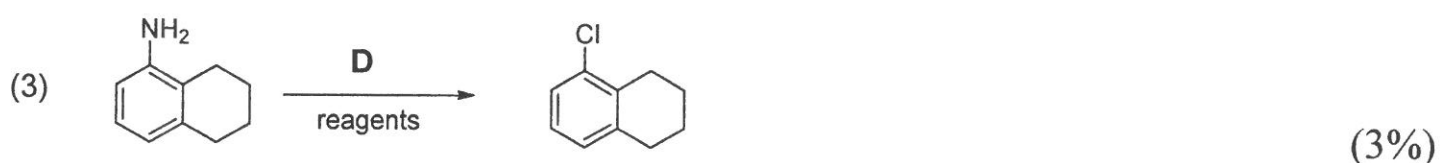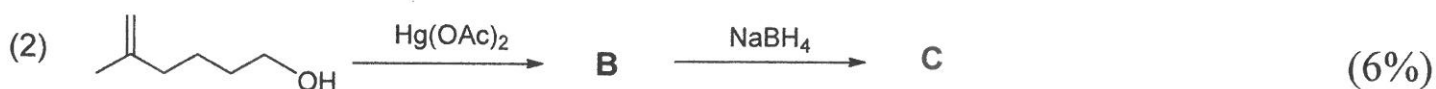**3.** A compound **B** with molecular formula $C_8H_{14}O_4$ exhibits a triplet at δ 1.3 (6 H), a singlet at δ 2.6 (4 H) and a quartet at δ 4.2 (4H) in its ^1H NMR spectrum. Its IR spectrum shows a strong absorption band near 1740 cm^{-1} and 1250 cm^{-1}. What is the structure for compound **B**? Rationalize your assignments including element of unsaturation and all spectra data. (10%)

科目：有機化學　　　　　　　　　　　　　　（全六頁，第五頁）

4. (30%) Predict what are the major product(s) or necessary reagents (**A** to **J**) for the following reactions. Include stereochemistry where applicable.

(1) $\xrightarrow[\text{E2}]{\text{NaOCH}_2\text{CH}_3}$ **A**　　　　　　(3%)

(2) $\xrightarrow{\text{Hg(OAc)}_2}$ **B** $\xrightarrow{\text{NaBH}_4}$ **C**　　　(6%)

(3) $\xrightarrow[\text{reagents}]{\textbf{D}}$ 　　　(3%)

(4) $\xrightarrow[\text{EtOH}]{\text{Na, NH}_3\,(l)}$ **E** $\xrightarrow[\text{(2)}\ \text{CHO}]{\text{(1) H}^+\ \text{(isomerization)}}$ **F** (Molecular formula $C_{10}H_{14}O_2$)　(6%)

(5) + $\xrightarrow{\text{heat}}$ **G**　　　(3%)

(6) $\xrightarrow{\text{peperidine/H}^+}$ **H** $\xrightarrow[\text{(2) H}_3\text{O}^+]{\text{(1) benzoyl chloride}}$ **I**　(6%)

(7) $\xrightarrow[\text{(2) H}_2\text{O}]{\text{(1) excess PhMgBr}}$ **J**　　　(3%)

（接下頁）

科目：有機化學　　　　　　　　　　　　　　（全六頁，第六頁）

5. (20%) Show how you would accomplish the following synthetic conversion in good yields. You may use any alcohols containing five or fewer carbon atoms, cyclopentanol, and any necessary solvents and reagents. Give the structures of intermediates and provide the required reagents. Be sure to indicate stereochemistry when applicable.

(1) (5%)

(2) acetylene ⟶ (5%)

(3) (5%)

(4) (5%)
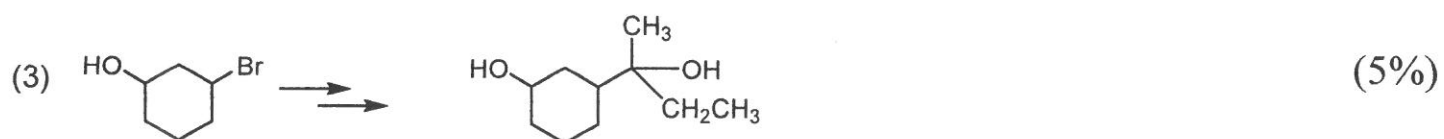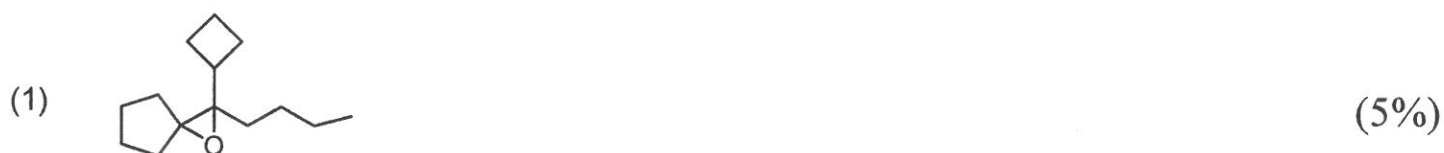
科目：分析化學　　　　　　　　　　　　　　　　（全一頁）

※可使用**一般**計算機（限僅具備＋、－、×、÷、％、√、MR、MC、M+、M-運算功能）

※繪圖限用黑色或藍色之鋼筆或原子筆作答

一、（總分 25 分）建構光譜儀所需的核心零組件包含：單光器(monochromator)、偵檢器(detector)、訊號處理與讀取裝置(signal processor and readout)、樣品槽(sample)與光源(radiation source)。請繪出紫外光－可見光光譜儀與螢光光譜儀的儀器設計中，上述零組件的空間排列方式，並比較兩種儀器設計的差異。（15 分）請繪圖說明光柵單光器的分光原理，與稜鏡單光器相比，最主要的優勢為何？（10 分）

二、（總分 25 分）檢量線(calibration curve)的建立為定量分析不可或缺的一環。請簡要描述以下三種方法取得檢量線的實驗流程：外標法(external standard method)、內標法(internal standard method)與標準添加法(standard addition method)，並比較其優缺點。（15 分）請討論上述哪種方法最適合應用於有限時間下，尿液樣品的藥物濫用篩檢。（10 分）

三、（總分 25 分）離子選擇性電極可藉由選擇性薄膜的選用與電極的設計應用於檢測溶液中的離子甚至是溶於水中的氣體分子。請繪製可檢測水中二氧化碳溶解度的薄膜電極的結構，並說明所使用的電極材料、電解液組成與感測薄膜特性。（15 分）請根據所涉及的化學方程式，平衡常數與電位響應公式說明此二氧化碳電極的工作原理。（10 分）

四、（總分 25 分）螢光探針因分析物的存在而發生螢光淬滅的現象已被廣泛的應用於檢測溶液中的重金屬離子，如：汞離子。然而，部分螢光探針的螢光強度極易受到酸鹼值的影響，而必須使用緩衝溶液來維持待測系統的酸鹼度。請列出相關的化學方程式與公式討論緩衝溶液如何維持溶液酸鹼度，並定義緩衝能力(buffer capacity)。（10 分）請繪圖說明緩衝溶液的組成如何影響緩衝能力。（5 分）利用螢光探針進行金屬離子的定量分析時，請討論緩衝溶液的使用對於分析金屬離子濃度可能造成的定量誤差來源。（10 分）

（試題隨試卷繳回）