教育部 111 年公費留學考試試題 127

科目：普通物理 **（全三頁，第一頁）**

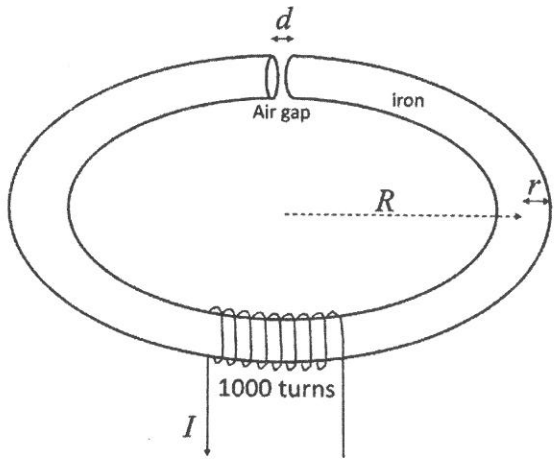
※以中文或英文作答均可，評分基準相同。

※可使用**工程**計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)。

1. A chamber is cooled by an air-conditioner that operates with reversible Carnot cycles. The air-conditioner consumes power P. The temperature inside the chamber is T_2, and the ambient temperature outside the chamber is T_1, $T_1 > T_2$. The heat transfer to the chamber through the walls is proportional to the temperature difference, with a proportional constant k. When the temperature is stabilized, express T_2 in terms of k, T_1 and P. (10%)

2. The temperature at the surface of the Sun is 5778K. If we treat the Sun and the Earth as black bodies,
   (a) what is the total power transfer from the Sun to the Earth by radiation? (8%)
   (b) what is the expected surface temperature of the Earth? (8%)
   The following parameters may be needed for your calculation: Radius of Sun = 696340 km; Radius of Earth = 6371 km; Distance from Sun to Earth = 149.6 million km. The Stefan-Boltzman law says that radiation heat transfer per unit area for a black body is $Q = \sigma T^4$, where T is the temperature of the object and $\sigma = 5.6697\times10^{-8}$ W/m^{2}K^4. The temperature of deep space is 2.7K, and its radiation can be neglected.

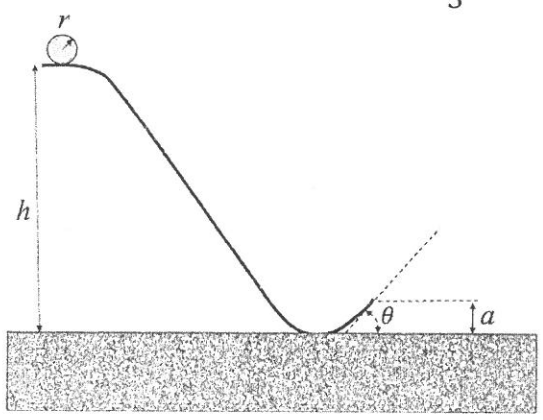
3. An electrical magnet is schematically shown in the attached figure. The magnet core is a ring of radius $R = 1$ m and circular cross section of radius $r = 10$cm, with an air gap of width d (=1cm). The solenoid wire winds 1000 turns around one side of the magnet core, and carries a current of $I = 5$A. The magnet is made of iron with a relative magnetic permeability of 5000 when it is not saturated. Estimate the magnetic field (in Tesla) in the air gap.
   Note: To solve this problem, assume the field is uniform in the air gap, and ignore the edge effect and field leakage. The relative magnetic permeability is defined as: $B = \mu_r\mu_0 H$, where μ_r is the relative permeability, and μ_0 is the absolution permeability of vacuum (or air). $\mu_0 = 1.257 \times 10^{-6}$ Henry/m. (10%)



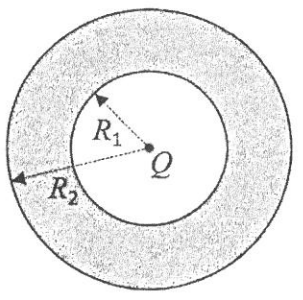
(接下頁)

科目：普通物理 （全三頁，第二頁）

4. A solid ball of radius r, initially at rest, rolls down without slipping a rail of height h and then shoots up where the rail turns upward. At the end point, the height of the rail is a and the tangent angle is θ. Find the maximum height that the ball can reach after leaving the rail. Note that the radius r should not be neglected, and the moment of inertia of a solid ball is $\frac{2}{5}mr^2$, where m is the mass of the ball. (10%)



5. A charge Q is located at the center of a hollow spherical shell with inner radius R_1 and outer radius R_2. The total charge of the shell is zero. What are the electric fields as functions of the distance to the center, r, under the following two conditions:
(a)The shell is made of conductive material. (8%)
(b)The shell is made of insulator with a dielectric constant (relative permittivity) ε_r. (8%)



6. A rocket in space (free of gravity) of total mass M is propelled by ejecting fuel at a constant rate of $\frac{dM}{dt} = \mu$ (note: $\mu < 0$). The fuel is ejected with a constant exhaust velocity v_e relative to the rocket.
(a)Write down the differential equation that determines the rocket's motion, i.e., what is $\frac{dv}{dt}$, where v is the velocity of the rocket, in terms of v_e, μ, and M. (8%)
(b)If the rocket has an initial mass M_0, solve the equation and find out what is the velocity change when the mass of rocket is reduced by ΔM. (6%)

(接下頁)

科目：普通物理 （全三頁，第三頁）

7. A 1 kg mass is suspended from a thread 55 cm long that can sustain a tension of 12.0N before breaking.
   (a)What is the frequency of small amplitude pendulum motion? (6%)
   (b)Find the maximum allowable amplitude (in angle) for pendulum motion of this system. (8%)

8. One arm of a Michelson interferometer is 50 cm long and is enclosed in a box that can be evacuated. The box initially contains air at 1atm, which is gradually pumped out. In the process, 621 bright fringes pass a point in the viewer. If the interferometer uses light with wavelength 480 nm in vacuum, what is the air's refractive index at 1 atm? (10%)

（試題隨試卷繳回）

教育部 111 年公費留學考試試題 128

科目：工程數學 （全一頁）

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※以中文或英文作答均可

1. (20%) Let $f(x) = x^2$, $-\pi \le x \le \pi$, be a 2π-periodic function.

(a) (10%) Find the Fourier series of f.

(b) (10%) Use the result in (a) and the Parseval's identity to show that

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.$$

2. (20%) Let y_1 and y_2 be any two solutions to the homogeneous part (*i.e.*, $f(t) = 0$) of the linear differential equation

$$y''(t) + p(t)\, y'(t) + q(t)\, y(t) = f(t). \tag{1}$$

Here $p(t)$, $q(t)$, and $f(t)$ are given functions of t for $t \ge 0$. The Wronskin $W(y_1, y_2)$ of these solutions is given by $W(y_1, y_2) = y_1 y_2' - y_2 y_1'$.

(a) (10%) Show that $W(t)$ satisfies the first order differential equation

$$W'(t) + p(t)\, W(t) = 0,$$

and find the solution of $W(t)$ with the initial condition W_0 at $t = 0$.

(b) (10%) If the Wronskin $W(y_1, y_2)$ is nonzero for all t, show that the general solution of (1) is

$$y(t) = c_1 y_1 + c_2 y_2 - y_1 \int \frac{f(z)\, y_2(z)}{W(y_1, y_2)}\, dz + y_2 \int \frac{f(z)\, y_1(z)}{W(y_1, y_2)}\, dz,$$

where c_1 and c_2 are constants.

3. (20%) Consider the system of linear ordinary differential equations

$$\frac{dY(t)}{dt} = AY(t) \quad \text{with} \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \tag{2}$$

(a) (10%) Find the eigenvalues and the eigenvectors of A.

(b) (10%) Find the general solution of (2).

4. (20%) Solve the heat equation

$$u_t = \varepsilon\, u_{xx}, \quad \varepsilon > 0,$$

in a semi-infinite line $x \ge 0$, satisfying the boundary conditions

$$u(0, t) = f(t), \quad t \ge 0, \qquad u(x, t) \to 0 \quad \text{as} \quad x \to \infty,$$

and the initial condition $u(x, 0) = 0$.

5. (20%) Consider the elliptic boundary value problem of the form

$$\begin{cases} u_{xx} + u_{yy} = 0, & \text{for} \quad x^2 + y^2 \le 1, \\ u(x, y) = x^2, & \text{for} \quad x^2 + y^2 = 1. \end{cases} \tag{3}$$

(a) (10%) Show that in polar coordinate with $(x, y) = (r\cos\theta, r\sin\theta)$ the boundary value problem (3) becomes

$$\begin{cases} u_{rr} + \dfrac{1}{r} u_r + \dfrac{1}{r^2} u_{\theta\theta} = 0, & \text{for} \quad r \le 1, \\ u(r, \theta) = \cos^2\theta, & \text{for} \quad r = 1. \end{cases} \tag{4}$$

(b) (10%) Find the solution of this problem.

（試題隨試卷繳回）

教育部 111 年公費留學考試試題 129

科目：材料科學 **(全二頁，第一頁)**

※可使用工程計算機(限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※以中文或英文作答均可，評分基準相同。

1. The typical yield strength and plane strain fracture toughness of aluminum and aluminum alloy 2024 (Al-4.4 Cu-1.5 Mg-0.6 Mn) are listed below.

Material	Yield Strength (MPa)	Fracture Toughness (MPa·$m^{1/2}$)
Aluminum	35	24
Aluminum alloy 2024	320	44

Draw schematically a stress-strain diagram for both materials. Explain briefly the increases in these mechanical properties of aluminum alloy in comparison to aluminum by proposing possible mechanisms. (20%)

2. Magnesium alloys have replaced engineering plastics for many applications since that they are comparably light, stiffer and more recyclable. Aluminum is the foremost additive in these alloys. In the Mg-Al system, an eutectic equilibrium between the Mg solid solution (β phase) with the maximum solubility of 12.6 wt.% Al and the compound $Mg_{17}Al_{12}$ (γ phase) with 43.0 wt.% Al exists at 437°C and 34.1 wt.% Al. Draw schematically the equilibrium microstructures for a Mg-6 wt.% Al alloy at the temperatures (1) between the liquidus and solidus lines, (2) between the solidus and solivus lines and (3) below the solivus line. (15%)

3. Glass-ceramics are commonly used as ovenware for their excellent resistance to thermal shock. Relate the performance with their microstructure. Describe the change in their viscosity with temperature when they are heated till being liquid in comparison with a normal glass. Indicate the annealing point, melting point, softening point, strain point and working point of a glass-ceramic on a schematic viscosity-temperature diagram. How can one reduce the softening point of glass-ceramics? (15%)

(接下頁)

4. The thermosetting styrene-butadiene rubber and the thermoplastic styrene-butadiene block copolymer are both elastomers derived from the same monomers. Which of them is recyclable? Why? Also explain why they possess different mechanical responses at elevated temperatures. The chemical formula of styrene and butadiene are $C_6H_5CH=CH_2$ and $(CH_2=CH)_2$ respectively. (15%)

5. The electrical conductivity (σ) of a *p*-type silicon increases, decreases and increases again with increasing temperature (T) at low, middle and high temperature ranges respectively. Explain these relations between σ and T with an aid of a logarithm plot. (15%)

6. Barium titanate $BaTiO_3$ has a perovskite crystal structure in which Ba atoms form a simple cubic lattice, and Ti and O atoms occupy the cube center and the centers of cube faces respectively above 120°C. Describe the piezoelectricity and ferroelectricity of $BaTiO_3$. Assuming the cube is closed packed, calculate the ratios between the ionic radii of Ba, Ti and O ions. (20%)

教育部 111 年公費留學考試試題 132

科目：波浪力學 **（全二頁，第一頁）**

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

一、(總分 40 分，每題 5 分)解釋名詞

(一) 風浪 (wind wave)

(二) 湧浪 (swell)

(三) 瘋狗浪 (rogue wave or freak wave)

(四) 近岸流 (nearshore current)

(五) 裂流 (rip current)

(六) 波譜 (wave spectrum)

(七) 黑潮 (kuroshio current)

(八) 波浪繞射 (wave diffraction)

二、臺灣東部海域離岸不遠的平均水深約 1000 公尺，試求海嘯的傳遞速度？(20 分)

三、根據能量守恆定律，波浪在粒徑為 D 的單位體積能量消散率表示為

$$\frac{1}{h}\frac{d(EC_g)}{dx} = -D_*(D) \qquad (1)$$

式中 h 為平均水深，x為離岸方向垂直海岸之座標，E 為單位面積波浪能量，ρ為海水密度，g為重力加速度，H為波高，C_g為淺水群波波速，分別表示為

$$H=\gamma h \qquad (2)$$

$$\mathrm{E} = \frac{1}{8}\rho g H^2 \qquad (3)$$

$$C_g = \sqrt{gh} \qquad (4)$$

式中 γ 為經驗係數。請根據式(1)至(4)，推導出平衡海灘斷面形式為下式：

$$h = Ay^{\frac{2}{3}} \qquad (5)$$

式中 A 表示如下：

$$A = \left[\frac{24D_*(D)}{5\rho g\sqrt{g}r^2}\right]^{\frac{2}{3}} \qquad (6)$$

(20 分)

（接下頁）

教育部 111 年公費留學考試試題 132

科目：波浪力學 （全二頁，第二頁）

四、波浪觀測數據波高和週期如下表所示，試求示性波高(significant wave height)和週期 (wave period)。(20 分)

編號	波高(m)	週期(s)
1	5.9	11.9
2	5.0	10.8
3	5.4	11.6
4	4.5	9.8
5	4.7	10.4
6	3.2	8.2
7	9.7	10.4
8	11.1	12.1
9	12.4	11.6
10	8.5	13.6
11	7.4	13.1
12	5.5	12.1

（試題隨試卷繳回）

教育部 111 年公費留學考試試題 133

科目：結構學 **（全三頁，第一頁）**

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

一、 如圖 1 所示，樑之 AC 段受一均變負載作用，CD 段受一集中負載作用，請繪出該樑之剪力圖及彎矩圖。(20 分)

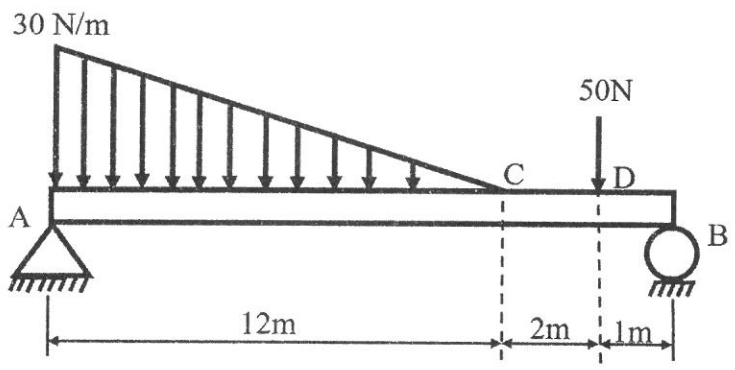


圖 1

二、 如圖 2 所示，此樑為兩懸臂樑由鉸鏈(hinge)相接至 C 點，並分別受 5kN 的集中負載作用在樑之 B 點、D 點，請畫出該樑之剪力圖以及彎矩圖。(20 分)

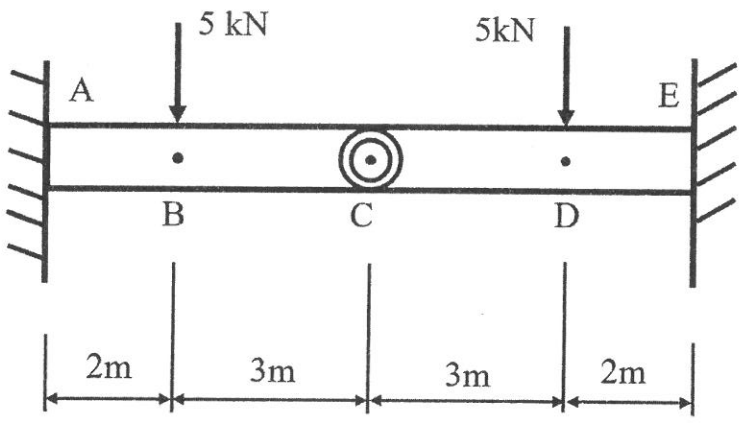


圖 2

（接下頁）

教育部 111 年公費留學考試試題 133

科目：結構學 **（全三頁，第二頁）**

三、 圖 3 為桁架(truss)結構，在 G 點受一 1000N 集中負載，在 H 點受一 600N 集中負載，請求出該圖中 DG、DF、BE 桁架所受之內力大小。(20 分)

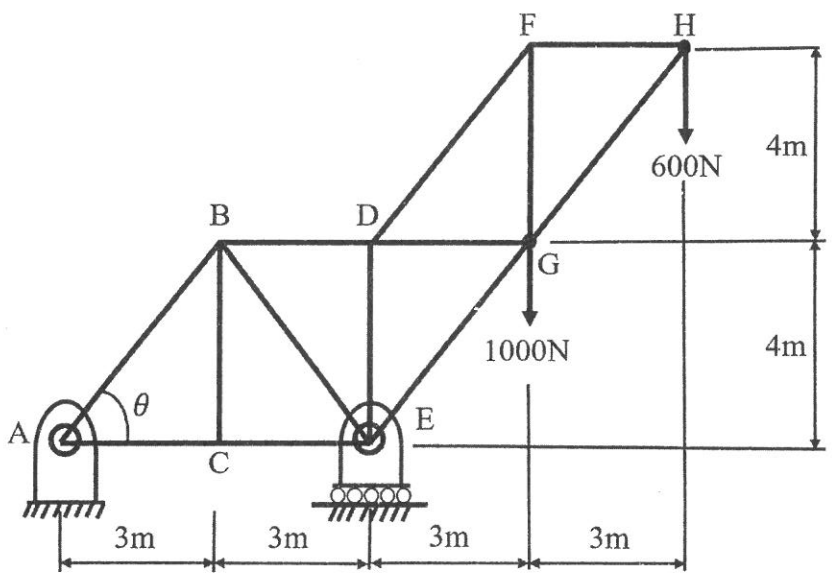


圖 3

四、 如圖 4 所示為桁架(truss)結構，受力分布如該圖所示，請求出該圖中 1 號桁架之內力。(20 分)

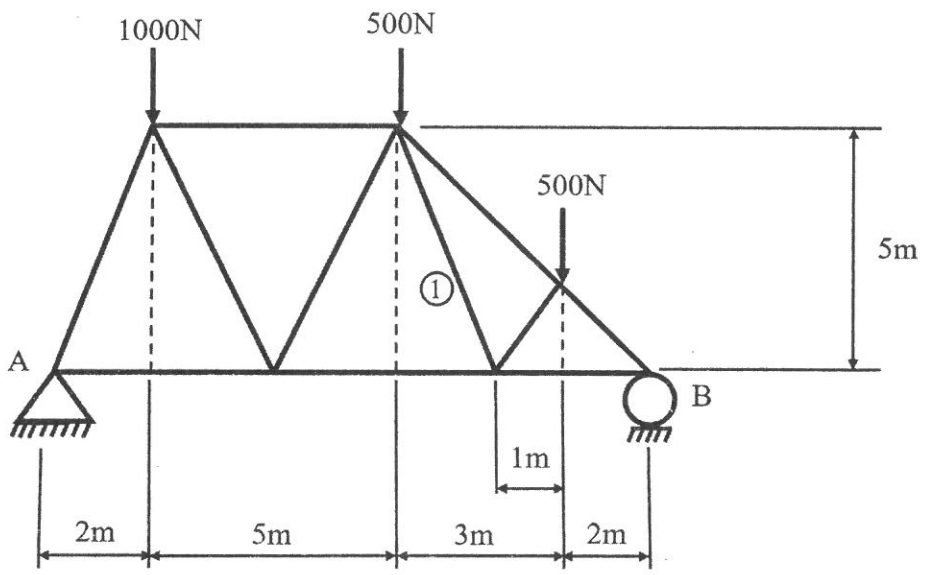


圖 4

（接下頁）

教育部 111 年公費留學考試試題 133

科目：結構學 （全三頁，第三頁）

五、 如圖 5 所示為桁架結構，在 A 點受 8 kN 集中負載，假設此桁架截面積 A 均為 200 mm^2、彈性模數 E 為 300 GPa，請使用卡氏定理(Castigliano's theorem)求出 A 點垂直位移量(Δ_A)。(20 分)

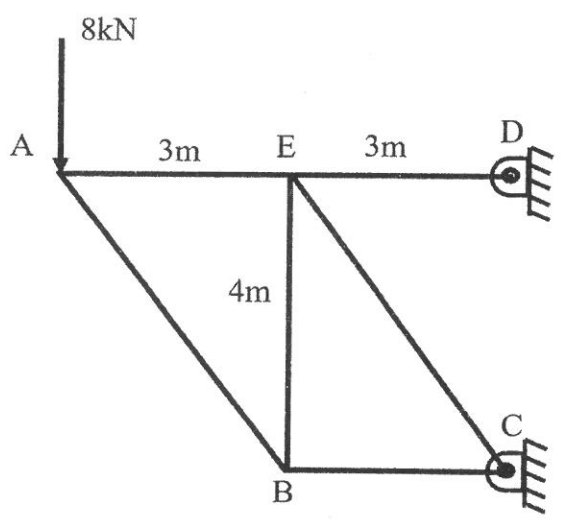


圖 5

（試題隨試卷繳回）

教育部 111 年公費留學考試試題　134

科目：船舶力學　　**（全一頁）**

※ 可使用**工程**計算機(限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

一、某船舶具長方體外形平浮於淡水（密度 1 g/cm^3）中，船長 10 m，船寬 4 m，船深 3m。該船舶質量分佈$m(x)$如下式所示：

$$m(x) = 6x - 0.6x^2 \qquad 0 \le x \le 10$$

其中x（單位 m）為船長方向座標，$m(x)$的單位為 ton/m。若船舶初始定傾高為 0.2 m，以及考慮船體僅受到浮力與重力作用（重力加速度 9.81 m/s^2），試求出以下船舶相關物理量：(總分 30 分)

（一） 船舶吃水。(5 分)

（二） 船體重心位置與水面的距離。(5 分)

（三） 船體最大剪力的位置及大小。(10 分)

（四） 船體最大彎矩的位置及大小。(10 分)

二、某船舶進行模型阻力實驗，根據 ITTC 1978 建議的方法，忽略船體表面粗糙度與空氣阻力的影響，假設波浪阻力係數無尺度效應且形狀因子為 1.14，重力加速度 9.81 m/s^2，試以平板阻力係數公式與下表推估該船舶實船阻力。(20 分)

$$C_f = \frac{0.075}{(\log Re - 2)^2}$$

其中C_f為平板阻力係數，Re為雷諾數。

實船					模型			
船速 (kn)	船長 (m)	浸水面積 (m^2)	海水密度 (g/cm^3)	海水黏度 (g/cm·s)	尺度	阻力值 (N)	淡水密度 (g/cm^3)	淡水黏度 (g/cm·s)
15.2	120	2263	1.025	0.014	1:20	34.26	1.000	0.01

三、若將船舶推進器視為一理想圓盤並忽略流體黏性，試藉由動量理論推導推進器理想效率η_I與其推力負荷係數C_T的關係。(15 分)

四、船舶螺旋試驗常用於量測船舶的特定運動性能，試說明船舶螺旋試驗目的與內容，並以圖示說明該運動性能與舵角、平擺角速度的關係。(15 分)

五、試說明船舶動穩度的意義以及如何評估船舶的動穩度。(10 分)

六、試說明船體振動的種類以及造成船體振動激振力的種類。(10 分)

（試題隨試卷繳回）

教育部 111 年公費留學考試試題 138

科目：熱流科學 **(全二頁，第一頁)**

※可使用**工程**計算機(限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

一、 一片初始為高均溫 T_0 的大平板瞬間浸入一個大的低恆溫(T_∞)液槽。板厚為 L，平板的熱傳導係數為 k_s，液體的熱傳導係數為 k_f，板與液體間的對流熱傳係數為 h。(總分 25 分)

(一) 關於此大平板降溫的暫態過程，你如何判定是否可採用塊狀熱容法(lumped capacitance method)？(5 分)

(二) 如果塊狀熱容法**適用**，請示意畫出從初始($t=0$)到極長時間($t\to\infty$)中平板內部溫度的變化過程。(5 分)

(三) 如果塊狀熱容法**不適用**，請示意畫出從初始($t=0$)到極長時間($t\to\infty$)中平板內部溫度的變化過程。(5 分)

(四) 針對塊狀熱容法**不適用**的狀況，寫出能描述平板溫度變化的能量方程式與邊界條件，其中需要的熱物理參數請自行採用並說明之。(10 分)

二、 (總分 25 分)

(一) 示意畫出流速為 U 的均勻流平行流經一大平板的層流速度邊界層區域，圖中設定平板前端為 $x=0$。並請說明此邊界層厚度$\delta(x)$的成長率與 x 的關係。(5 分)

(二) 若流體的普朗特數(Pr)為 8，請與前面$\delta(x)$曲線圖中同步畫出厚度為$\delta_t(x)$的熱邊界層區作比較。並請討論$\delta(x)$與$\delta_t(x)$差異的物理原因。(10 分)

(三) 若平板長度足夠讓過渡區與紊流區發生，請寫出通常訂為紊流區發生準則(criterion)的表示式，並比較紊流狀況下$\delta(x)$與$\delta_t(x)$的大小。(10 分)

三、 將航空用渦輪噴射引擎(turbojet engine)的熱力循環近似為空氣標準布雷登循環(air-standard Brayton cycle)。(總分 20 分)

(一) 示意畫出一渦輪噴射引擎的結構示意圖，須包括主要單元(如壓縮機等)。(5 分)

(接下頁)

(二) 分別畫出空氣標準布雷登循環的 *p-v* 圖與 *T-s* 圖，並在各過程(process)中標示該過程的熱交換狀況。(7 分)

(三) 針對其 *T-s* 圖，說明各過程為何可做如此假設，並進而畫出較接近真實過程的 *T-s* 圖。(8 分)

四、 空氣之氣體常數 $R = 0.287$ kJ/(kg • K)、熱容比 $k = 1.4$。(總分 30 分)

(一) 何謂似平衡過程(quasi-equilibrium process)？其與平衡過程(equilibrium process)有何不同？為何工程上需要定義出似平衡過程？(7 分)

(二) 以一汽缸中之活塞的往復運動來看，假設活塞在 10 cm 的汽缸長度中以每分鐘 3000 轉(3000 rpm)作往復運動，汽缸中空氣氣溫維持 1000°C，就汽缸中之壓力變化而言，此往復運動過程是否可視為似平衡？請根據簡易的估算回答。(8 分)

(三) 在超音速流中，一鈍體前方會產生一正震波(normal shock wave)，請解釋其發生原因。(7 分)

(四) 正震波的上下游壓力有何差異？正震波的發生是否可視為一似平衡過程？請解釋原因。(8 分)

教育部 111 年公費留學考試試題 140

科目：普通物理及普通化學 （全二頁，第一頁）

※以中文或英文作答均可，評分基準相同。

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

1. A 60 kg person rides a 10 kg bicycle on a smooth road in a windless day. On a 5° downhill section, she slides without pedaling, and reaches a terminal speed of 40 km/hr. Considering only the gravity, the pedaling power, and the air resistance (which is proportional to the square of the speed), what pedaling power she has to exert if she wants to reach a speed of 20 km/hr when the road is 3° uphill? (10%)

2. Hot water of mass m and temperature T_1 is mixed with same amount of cold water with temperature T_2. Assume a constant specific heat c, find the change of the entropy. (10%)

3. A coil is placed with its plane perpendicular to a uniform magnetic field. It is then flipped by 180°. The coil is connected to an instrument that measures the total charge Q that flows during the flipping. If the coil has N turns, area A, and total resistance R. What is Q in terms of N, A, and R? (10%)

4. An ambulance driving at 100 km/hr is sounding a siren of 600 Hz. What is the frequency sensed by a person in front of the ambulance and moving with 10 km/hr towards it? Assume the sound speed is 343 m/s. (5%)

5. (15%)A solid sphere of radius R contains a uniform volume charge density ρ.
   (a) What is the electric field at a distance r ($r > R$) from the center of the sphere. (5%)
   (b) What is the total electrostatic energy of this configuration? (10%)

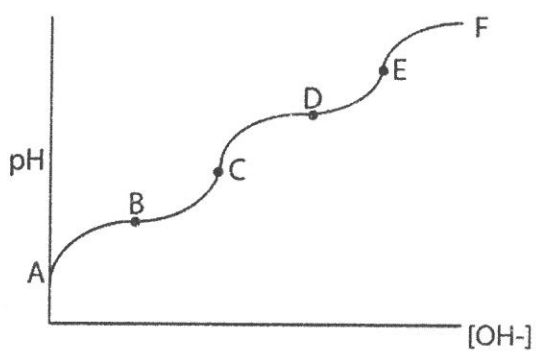
（接下頁）

6. (25%) Consider the following information about the 0.10 M diprotic acid (H_2A) in aqueous solution.

$H_2A \leftrightarrow HA^- + H^+$ $\quad pK_{a1} = 6.0$ ($K_{a1} = 1.0 \times 10^{-6}$)

$HA^- \leftrightarrow A^{2-} + H^+$ $\quad pK_{a2} = 9.0$ ($K_{a2} = 1.0 \times 10^{-9}$)

The titration curve for H_2A with standard 0.10 N NaOH is shown below:



(a)Calculate the pH values at point A, B, C, D, and E. (15%)

(b)What are the most abundance species at point B, C, D, E, and F? (10%)

7. (25%) A solution contains 1.0×10^{-3} M for IO_3^-, I_3^-, I^-, and pH 7.00 buffer at 25°C. Given the half reactions and each standard reduction potential shown in (1) and (2) in a solution.

(1) $2IO_3^- + I^- + 12H^+ + 10e^- \leftrightarrow I_3^- + 6H_2O$ $\quad E° = 1.210$ V

(2) $I_3^- + 2e^- \leftrightarrow 3I^-$ $\quad E° = 0.535$ V

(a) Obtain and balance the redox reaction (3) by half reactions (1) and (2). (4%)

(b) Predict whether the reaction direction go forward or not for the reactions (1), (2), and (3) as given condition by using the Nernst Equation: (12%)

$$E_{cell} = E^0 - \left(\frac{0.0592\ V}{n}\right) \log Q$$

(c) Calculate the pH in equilibrium. (5%)

(d) What is the value of log$K$ (K is the equilibrium constant) for the reaction in this solution? (4%)