教育部 111 年公費留學考試試題 117

科目：計算機結構 **(全三頁，第一頁)**

※可使用工程計算機(限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※可用中文或英文回答問題

1. (10 pts.) Consider two different implementations, P1 and P2, of the same instruction set architecture. There are five classes of instructions, A, B, C, D, and E. The clock rate and the CPI (cycles per instruction) of each implementation are given in the following table.

	Clock Rate	CPI_A	CPI_B	CPI_C	CPI_D	CPI_E
P1	2.0GHz	1	2	3	5	4
P2	1.8GHz	3	3	3	3	3

   (1) Given a program with 10^5 instructions divided into classes as follows: 10% Class A, 15% Class B, 40% Class C, 20% Class D, and 15% Class E. Which implementation (P1 or P2) is faster? (5 pts.)

   (2) Following (1), you are trying to improve P1 by saving 20% in the execution time. However, this leads to an increase of 15% of the average CPI. What clock rate should we have to set to achieve this time reduction? (5 pts.)

2. (15 pts.) A 1 GHz machine can perform "jump" instruction with 1 clock cycle, "branch" instruction with 3 cycles, "arithmetic" instruction with 2 cycles, "multiply" instruction with 5 cycles and "memory access" with 4 cycles. A program called "James" has 10% jumps, 10% branches, 50% arithmetic, 10% multiply, and 20% memory access.

   (1) What is the CPI of this program on this machine? (5 pts.)

   (2) Now, if "James" executes 10^7 instructions on this machine. What is its execution time? (5 pts.)

   (3) Following (2), a 6-cycle multiply-add instruction is implemented in this machine and it can combine an arithmetic and a multiply instruction. 50% of the multiply instructions can be transferred to this multiply-add instruction in "James". What are the new CPI and the execution time if the clock period remains the same? (5 pts.)

(接下頁)

3. (35 pts.) About memory hierarchy
   (1) Describe the reason of using memory hierarchy and how it works. (5 pts.)
   (2) Given cache, TLB, page table, main memory and the secondary storage (disk), describe the process of a memory access in a systematical way. (10 pts.)
   (3) Describe the positive and negative effects of increasing (a) cache size, (b) associativity and (c) block size to the overall performance of memory access. (10 pts.)
   (4) Find the average memory access time (AMAT) with a 1 ns clock, a miss penalty of 20 clock cycles, a miss rate of 0.08 misses per instruction, and a cache access time (including hit detection) of 1 clock cycle. Assume that the reading and writing miss penalties are the same and ignore other writing stalls. (5 pts.)
   (5) As in (4), suppose we can improve the miss rate to 0.05 misses per reference by doubling the cache size. This causes the access time to increase to 1.8 cycles. Using the AMAT as the metric, please determine if this is a good trade-off. (5 pts.)

4. (20 pts.) For the cache memory design:
   (1) Please design a 2-way set-associative cache of 8K bytes data. The block size is 2 words with 4 bytes in each word. Assume that the total cacheable physical memory is 16MB bytes. Please show the physical address format (in the order of tag, index and offset from MSB to LSB) and then the block diagram of the design, including tags, data and valid bits. (10 pts.)
   (2) Continue (1). Here is a series of physical address references given as byte addresses:

(接下頁)

Label	Memory address
A	0x245625
B	0x245627
C	0x245628
D	0x321620
E	0x322623
F	0x245627
G	0x245630
H	0x322622

Assume that the cache is initially empty and LRU (Least Recently Used) replacement is adopted. Determine each reference (A~H) in the list as a hit or a miss and show the final content of the cache (i.e. the non-empty set number and the stored address range). (10 pts.)

5. (20 pts.) Briefly explain the following terms or answer the questions.
   (1) Dynamic scheduling. (5 pts.)
   (2) Loop unrolling. (5 pts.)
   (3) Compare super-scalar processors and VLIW processors. (10 pts.)

教育部 111 年公費留學考試試題 118

科目：演算法 **（全二頁，第一頁）**

※以中文或英文作答均可，評分基準相同。

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

1. (20%)

Given a set of n real number elements. Design an algorithm to find the largest two elements among them. Your algorithm must use at most $3/2n-2$ element comparisons. Assume n is a power of 2.

(a) Give a high-level description of your algorithm and explain the correctness of your algorithm. (10%)

(b) Give the recurrence to count the number of comparisons and solve this recurrence. (10%)

2. (20%)

(a) An n elements array contains all but one of the integers from 1 to $n+1$. Given the best algorithm for determining which number is missing if the array is sorted, and analyzed its asymptotic worst-case running time. (10%)

(b) Given two arrays of real number elements with size m and n each. Assume m is much smaller than n, design an O(n log m) time algorithm to check whether these two arrays are disjoint. (10%)

3. (20%)

A vertex cover of a graph $G=(V,E)$ is a set $S \subseteq V$ such that for each edge $e \in E$, at least one endpoint of e is in S. The vertex cover problem is to find the smallest size vertex cover of G.

(a) Show how to formulate the vertex cover problem as an Integer Linear Program (ILP) problem. (10%)

(b) Since solving ILP is NP-hard, Linear Programming (LP) is adopted to solve ILP vertex cover first, then the solution is rounding to integer solutions. Show that this LP rounding approach is a 2-approximation algorithm. (10%)

（接下頁）

4. (20%)

The decision version of the 0-1 knapsack problem is defined as follows. Given two positive integers V and W, and n objects $o_1, \dots, o_n$, where o_i has value v_i and weight w_i. Is there a subset S of the objects such that $\sum_{o_i \in S} w_i \leq W$ and $\sum_{o_i \in S} v_i \geq V$?

(a) Show that 0-1 knapsack $\in NP$. (3%)

(b) Show that 0-1 knapsack problem is NP-complete by a reduction from the Partition problem which is already known to be NP-complete. The Partition problem is defined by given a set A of positive integers $a_1, \dots, a_n$ such that $\sum_{i=1}^{n} a_i$ is even , and ask if there is a subset B of A such that $\sum_{a_i \in B} a_i = \frac{1}{2}\sum_{i=1}^{n} a_i$ (14%)

(c) Having designed a $O(nW)$ time dynamic programming algorithm for 0-1 knapsack, someone claims that it is polynomially solvable, thus $P = NP$. What is wrong with this argument? (3%)

5. (20%)

Let T be a weighted tree with a function $w: E \to R^+$. The length of a path $p = (v_0, v_1, \dots v_r)$ is defined to be $\sum_{i=0}^{r-1} w(v_i, v_{i+1})$.

(a) Design a linear time algorithm to find the longest path in this tree. (10%)

(b) Given the recurrence of worst case running time and solve this recurrence asymptotically. (10%)

科目：計算機程式 （全二頁，第一頁）

※以中文或英文作答均可，評分基準相同。

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)。

1. (5%) (a) What are printed by the following C/C++ program?

```
int a=14, b=24;
printf("%d %d\n", a&(a-1), b&(b-1)); // &: bitwise AND
```

(10%) (b) Please describe the purpose of `x&(x-1)` for any positive integer `x`.

2. (5%) (a) There is a C function `func( )` as follows. Suppose that the input value of `W` for `func( )` is always a prime number. In addition, the value of `W+2` is also a prime number. What are printed by `func( )`?

```
void func(int w)
{ int x=w % 6;  int y=(w+1) % 6;  int z=(w+2) % 6;
  printf("%d  %d  %d \n", x, y, z); }
```

(10%) (b) Show that your answer in (a) is always correct.

3. (15%) What are printed by the following C/C++ program?

```
int a[ ]={ 41, 45, 43, 47, 44, 56};
int func(int n) {
  if (n==0) {
    printf("n=%d f=%d \n", n, a[0]);
    return (a[0]);
  }
  else if (a[n] >= func(n-1)) {
    printf("n=%d f=%d \n", n, a[n]);
    return (a[n]);
  }
  else {
    int q=func (n-1);
    printf("n=%d f=%d \n", n, q);
    return (q);
  }
}
int main( ) {
  func(5);
}
```

(接下頁)

4. (15%) Suppose the values of A, B, C, D are 1, 2, 3, 1, respectively. A permutation can be formed by any number of these four symbols (A, B, C, D). The total value of a permutation is the sum of all values of the used symbols. For example, the total values of some permutations are shown: A=1, B=2, C=3, D=1, BBC=7, BCB=7, BCDA=7, AABAADCDDD=13. The total value of a permutation is denoted as n. Let $g(n)$ denote the number of permutations whose total values are n. Then $g(1)=2$, since there are 2 permutations with total value 1: A, D. $g(2)=5$ is obtained from the 5 permutations: AA, AD, DA, DD, B. $g(3)=13$ is obtained from the 13 permutations: AAA, AAD, ADA, ADD, DAA, DAD, DDA, DDD, BA, BD, AB, DB, C. In addition, $g(0)=1$ is set as the initialization. Please derive the recurrence formula for calculating $g(n)$, $n\geq 4$.

5. (5%) (a) Please give the definition of an AVL balanced binary tree.

   (15%) (b) Write a recursive function (with C/C++, JAVA, or Python) to check whether a given binary tree is an AVL tree or not. It is assumed that the root of the tree is the input to the function. You can make any other assumption or define any parameter of your function. Please point out your programming language.

6. A subsequence of a string X is obtained by deleting zero or more characters from X. A string X is a palindrome if it is the same for reading X from the left to the right and from the right to the left. Y is the longest palindrome subsequence (LPS) of a string X if Y is a subsequence of X, and Y is a palindrome with the maximal length. For example, suppose X=abcbabba. Then abbabba, abcbabba, acbba are three subsequences of X. In addition, the LPS of X is abbabba with length 7. As another example, the LPS of abccdbe is bccb with length 4.

   (15%) (a) Please design a dynamic programming (DP) method to calculate the LPS length of a given string X. You need to write out only the DP formula, not the complete program. Only the LPS length (not the LPS content) is required to answer. You can make any assumption and define any symbol for solving the problem. Note that the time complexity of your method should be no more than $O(n^2)$, where n denotes the length of X.

   (5%) (b) Please analyze the time complexity of your method, and represent the complexity with the O( ) notation.

教育部 111 年公費留學考試試題　120

科目：線性代數　（全一頁）

※ 可使用**工程**計算機（限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-、三角函數、對數、指數運算功能）

※ 以中文或英文作答均可，評分基準相同。

※ 推導或證明過程必須明確清楚，否則將酌予扣分或不給分。僅有答案而無推導或證明過程者，將不予給分。

※ Notation: R^n denotes the set of n-dimensional real vectors. For a given matrix A, A^T denotes its transpose.

1. (20%) Consider a 4×4 matrix A and the linear equation system $Ax = b$ whose augmented matrix $[A \mid b]$ has the reduced row echelon form

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & -2 \\ 0 & 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(a) (10%) Find a basis of the null space of A.

(b) (10%) Express b in terms of the second and the fourth columns of A.

2. (20%) Let A be an $n \times m$ real matrix whose columns are linearly independent.

(a) (10%) Prove that $A^T A$ is invertible.

(b) (10%) Find the matrix P that projects any R^n vector to the column space of A.

3. (20%) For the given data points (x_1, y_1), (x_2, y_2), $\cdots$, (x_n, y_n), find the least square circle that fits the n data points. That is, the circle minimizes the norm of the residual vector $\mathbf{e}$, where the component $\mathbf{e}_i = r^2 - d_i^2$, r and d_i represent the radius of the circle and the distance between (x_i, y_i) and the center of the circle, respectively.

4. (20%) Consider the vector subspace V of $C[-\infty, +\infty]$ spanned by functions $e^t \sin(t)$ and $e^t \cos(t)$. Define the integration operator $I: V \mapsto V$ as follows

$$I(f)(t) := \int_{-\infty}^{t} f(\tau) d\tau$$

Derive the matrix representation of I with respect to the basis $\{e^t \sin(t), e^t \cos(t)\}$。

5. (20%) Consider a two-state Markov process $x_{k+1} = Ax_k$, where

$$A = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$$

Suppose the initial state is $x_0 = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$. Find the steady-state vector that the process converges to.

（試題隨試卷繳回）

教育部111年公費留學考試試題 121

科目：計算機概論 **(全二頁，第一頁)**

※可使用工程計算機(限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※以中文或英文作答均可，評分基準相同。

1. Memory in a C/C++/Java program can either be resided in a data region, or allocated on a stack or a heap. Given a short C/C++ code on right. (25 points)
   a) From OS point of view, variables can reside in data segment, stack, and heap. Give the memory allocation purposes respectively. How are they managed?
   b) Give the address region for A, B, C, and *C, when **func()** is called.
   c) What happens if **func()** is finished and B[5] is written?

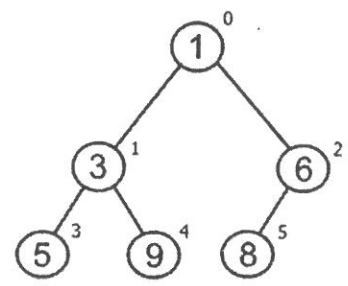
```
int A[1024];
int *func() {
    int a;
    int n = 2048;
    int B[n];

    int *C = new int[128];
    return C;
}

B = func();
B[5] = 10;
```

2. Multi-Threading is one of the important features in an operating system. (25 points)
   a) Give key at least 3 differences between processes and threading.
   b) Multithreading can be kernel-level or user-level threading. Please give the key differences. Also, compare the advantages and disadvantages of kernel-level or user-level multithreading respectively.

3. Memory hierarchy is a very important design in computer systems. (25 points)
   a) Give a brief explanation how L1, L2, L3, main memory, TLB, and virtual memory are organized in contemporary computers.
   b) What key ideas are behind the memory hierarchy to improve system performance?
   c) What is TLB? Why do we need a TLB for virtual memory?

(接下頁)

4. Heap is a tree data structure and can be stored in memory using links. An array can be used to represent a binary heap as shown on right. (25 points)



1	3	6	5	9	8
0	1	2	3	4	5

   a) If a heap is a complete binary tree, we can easily store it using a scalable array. If the left child of node ***i*** is mapped to ***Left(i)=2*i+1***. Please give the mapping of its right child node and parent node respectively.
   b) Write a sequence code of BinaryMinHeap, which can give the minimal number on a series of numbers stored in an array.
   c) What is the time complexity of inserting a new node in the above *N*-node BinaryMinHeap? Also, what complexity will it cost if we use BinaryMinHeap to sort *N* numbers?

(試題隨試卷繳回)

教育部 111 年公費留學考試試題 122

科目：計算機網路 （全一頁）

※以中文或英文作答均可

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

一、 Protocol (總分 30 分)

(一)簡述在 IPv4 與 IPv6 個別網路中如何做到即插即用(Plug & Play)。(15 分)

(二)簡述 RTP 與 RTCP 如何進行網路傳輸服務品質(Quality of Service) 控制。(15 分)

二、 Web & HTTP (總分 25 分)

(一)請解釋 HTTP 2.0 中 multiplexing 意義與功用為何？主要是用來解決 HTTP 1.1 中的何種問題？(15 分)

(二)IETF RFC 9309 REP (Robots Exclusion Protocol)的用途為何？請簡述其運作方式。(10 分)

三、 TCP Protocol (總分 45 分)

(一)為何 TCP 在建立連線時需要做 Three-way Handshake？如果在 Three-way Handshake 中最後一次 ACK 封包遺失會發生何事？(15 分)

(二)何謂 SYN flood 攻擊？如何防禦？(15 分)

(三)TCP flow control 如何避免接收端 buffer overflow？(15 分)

（試題隨試卷繳回）

教育部 111 年公費留學考試試題 123

科目：資訊安全概論 （全一頁）

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)。

一、 試簡要解釋下列資安名詞的意義。(總分 25 分)

(一)零時差攻擊(zero-day attacks) (5 分)

(二)零信任架構(zero trust architecture) (5 分)

(三)零知識證明(zero-knowledge proof) (5 分)

(四)紅隊演練(red team assessment) (5 分)

(五)APT 攻擊(advanced persistent threat) (5 分)

二、 瀏覽網頁使用的 HTTPS 通訊協定名字中的 S 代表 TLS (Transport Layer Security)協定，(總分 25 分)

(一)這個 TLS 協定的加密功能同時使用對稱式密碼法與非對稱式密碼法，兩者如何搭配？這樣做的好處是什麼？(15 分)

(二)TLS 的另一個重要功能是提供 X.509 憑證。這個憑證帶來的好處是什麼？又網頁瀏覽器如何驗證網頁伺服器憑證的真偽？(10 分)

三、 3DES (Triple DES)拉長了 DES 密碼法的金鑰長度，讓 DES 至今仍被使用於低階的加密系統。(總分 25 分)

(一)3DES 的作法為何？為何 3DES 可以與 DES 相容？(10 分)

(二)DES 的有效金鑰長度為 56 位元，3DES 的有效金鑰長度是多少？為何不是 56 位元的 3 倍？(10 分)

(三)AES (Advanced Encryption Standard)是接替 DES 的標準，其可能的有效金鑰長度為多少位元？(5 分)

四、 資安的要素包含機密性、完整性與不可否認性(non-repudiation)。以下的密碼學相關技術提供了以上哪些要素？並請簡述如何提供。(總分 25 分)

(一)PKI (Public Key Infrastructure) (9 分)

(二)SSH (Secure Shell) (8 分)

(三)PGP (Pretty Good Privacy) (8 分)

（試題隨試卷繳回）

教育部 111 年公費留學考試試題 124

科目：離散數學 （全一頁）

※可使用**工程**計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※可使用中文或英文作答

1. [10%] How many subsets of size 5 from a set that contains n elements?

2. [10%] We flip a coin for 15 times and obtain a result of x times of coin facing up and y times of coin facing down. What is the most likely outcome of (x, y)? Apparently, we have $x + y = 15$. What is the probability to have such outcome (x, y)?

3. [15%] Which series is bigger, $\sum_{n=1}^{\infty} a_n$ or $\sum_{n=1}^{\infty} b_n$? We have:

$$a_n = \frac{1}{n(n+1)}$$

and

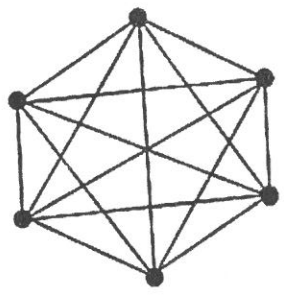
$$b_n = \frac{1}{2^n} .$$

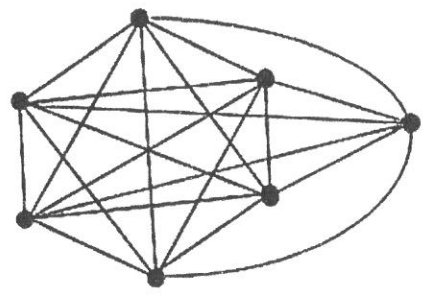
4. [15%] If both $a + b$ and ab are even, then both a and b are even. Prove or disprove the statement.

5. [15%] Solve the recurrence equation given by:

$$a_{n+2} = 2a_n + a_{n+1}, a_0 = 0, a_1 = 1.$$

6. [15%] Which of the following graphs, K_6 (complete graph with 6 vertices) or K_7 (complete graph with 7 vertices) has closed Euler walk? Explain your answer as clear as possible. That is, in closed Euler walk, we allow each edge to be visited exactly once, and the initial vertex and ending vertex are the same. Can we modify the graph(s) to make it/them possible to contain a closed Euler walk?





7. [20%] The following is a recursive definition of trees:

Def 1.

A tree consists of a root, and zero or more subtrees $T_1, T_2, \ldots, T_k$. Also, there is an edge from the root to the root of each subtree.

Now someone modifies the definition and give the following alternative one:

Def 2.

A tree consists of a root, and one or more subtrees $T_1, T_2, \ldots, T_k$. Also, there is an edge from the root to the root of each subtree.

Can you describe what will happen if we choose the second one as the definition instead? What trees can be generated by the two definitions?

（試題隨試卷繳回）

教育部 111 年公費留學考試試題 125

科目：半導體物理 **(全一頁)**

※可使用工程計算機(限僅具備＋、－、×、÷、%、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

※以中文或英文作答均可，評分基準相同。

1. (20%) For a p-n junction diode,
   (1) prove that the Fermi-level (E_F) is constant throughout the sample under thermal equilibrium condition. ($P=n_i\exp((E_i-E_F)/kT)$, where n_i is the intrinsic concentration, E_i is the quasi Fermi level). (10%)
   (2) when apply a forward bias and a reverse bias respectively, is the current diffusion or drift? Why? (10%)

2. (20%) For an Impact Ionization Avalanche Transit-Time (IMPATT) diode,
   (1) describe why it can be used as a high power amplifier? (10%)
   (2) how to generate a negative differential resistance? (10%)

3. (30%) Please design a heterojunction bipolar transistor (HBT),
   (1) plot its energy band diagram under thermal equilibrium. (10%)
   (2) what are its advantages when compare to homojunction bipolar transistor? (10%)
   (3) why the offset voltage exists in an HBT? how to improve it? (10%)

4. (30%) For a high electron mobility transistor (HEMT),
   (1) how can it form two-dimensional electron gas? (10%)
   (2) how can it achieve high electron mobility? (10%)
   (3) how to form an enhancement mode HEMT? (10%)

(試題隨試卷繳回)

教育部 111 年公費留學考試試題 126

科目：電子學 （全二頁，第一頁）

※以中文或英文作答均可，評分基準相同。

※可使用工程計算機(限僅具備＋、－、×、÷、% 、√、MR、MC、M+、M-、三角函數、對數、指數運算功能)

1. (20%) For the oscillator circuit show in Fig. 1 and assuming an ideal op amplifier,
(a) Find the oscillation frequency.
(b) Find the minimum value of R_2/R_1 to achieve oscillation.

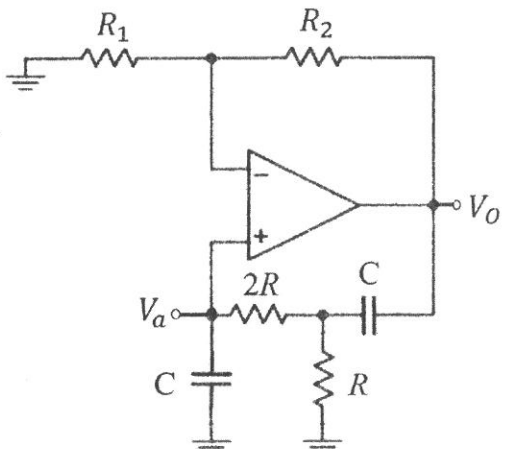


Fig. 1

2. (24%) Consider the common-base amplifier of Fig. 2. Assume $\beta >> 1$, and calculate
(a) the dc bias current I_E,
(b) the input resistance R_{in},
(c) the voltage gain v_o/v_s, and
(d) the maximum allowable amplitude of v_s.

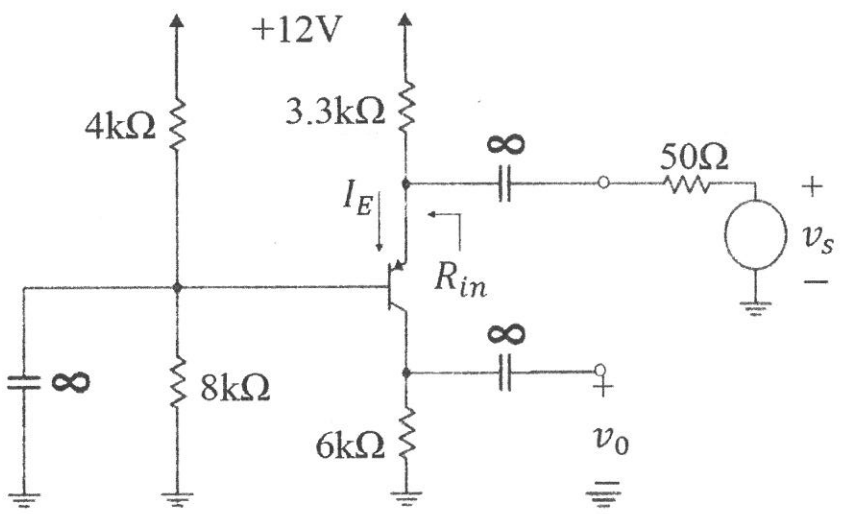


Fig. 2

（接下頁）

3. (32%) Consider the following circuit in Fig. 3. Assume at time $t = 0$, $V_X = 5$V.

(a) What is the collector current I_C of Q1 at $t = 0$? Please check the value of V_{BE}.

(b) What is I_C when $V_X = 1$V?

(c) If Q1 has an internal collector resistance $r_C = 200\Omega$, will Q1 saturate when $V_X = 1$V?

(d) Using the average current of I_C at $V_X = 5$V and 1V, calculate the time required for V_X to go from 5V to 1V.

where for M1 : $\mu_n C_{ox} = 40\ \mu A/V^2$, $W/L = 1$, threshold voltage $V_{THN} = 0.7$ V;

for Q1: $\beta = 50$, $I_S = 10^{-15}$A, $V_T = kT/q = 25$ mV, $I_C = I_S[exp(V_{BE}/V_T)-1]$ (in forward active region)

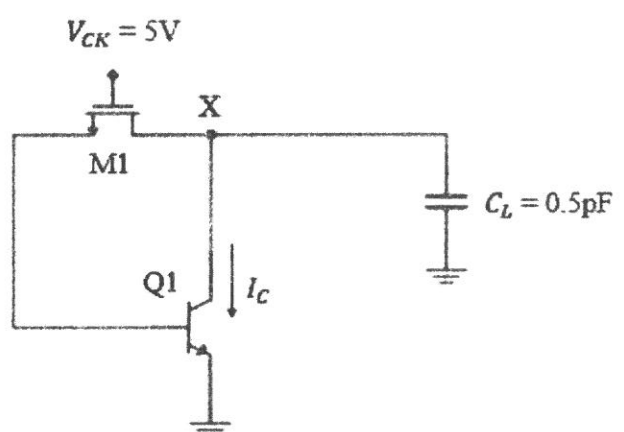


Fig. 3

4. (24%) Shown below in Fig. 4 is an ECL/MOS level converter, and the voltage level of A and $\bar{A}$ as a function of time.

(a) Explain the role of the diode-connected bipolar transistor Q3 in this circuit.

(b) Calculate the maximum and minimum voltages at nodes X and Y.

(c) Calculate the maximum and minimum voltages at node Z.

M1, M2 : $\mu_p C_{ox} = 25\ \mu A/V^2$, $W/L = 3/2$, threshold voltage $V_{THP} = -0.7$ V.

Q1, Q2, Q3: $\beta = 50$, $I_S = 10^{-18}$A, $V_T = kT/q = 25$ mV, $I_C = I_S[exp(V_{BE}/V_T)-1]$ (in forward active region)

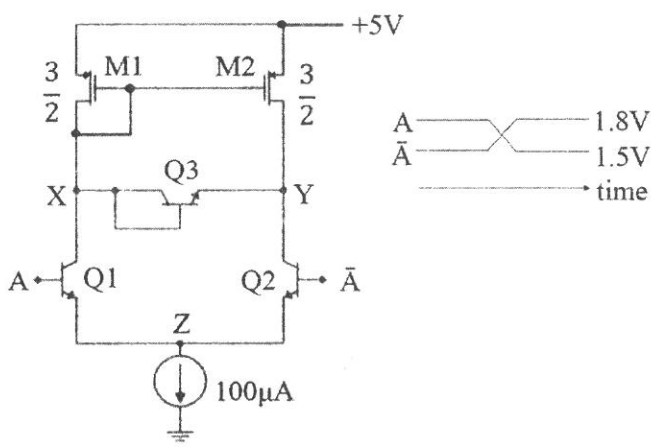


Fig. 4

（試題隨試卷繳回）